Critical Thinking in Pumping Test Interpretation

The interpretation of pumping well drawdowns

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Overview

The drawdowns in a pumping well can and should be interpreted for every pumping test. Key to the reliable interpretation of the drawdown data from a pumping well is an appreciation that the drawdowns in a pumping well reflect more than just the effects of head losses in the formation. In these notes the interpretation of pumping well drawdowns is built up gradually in complexity. The notes begin with the characterization of the components of the drawdown in a pumping well. Models are then discussed to support the inference of well characteristics and aquifer properties.

Outline

1. Definition of the drawdown in a pumping well

The drawdown in a pumping well is defined as the difference between the water level in a well under nonpumping conditions and the water level observed in the well when it is pumping.[1] The drawdown is illustrated schematically in Figure 1, but the definition is slightly more general than what is suggested in the figure. The non-pumping level in an aquifer is never a completely flat surface, nor will it remain constant through time. Data from a pumping test conducted in Portland, Oregon, are shown in Figure 2 to highlight that the drawdown is correctly interpreted as the difference between the pumping level and the level that would be observed in the absence of pumping.

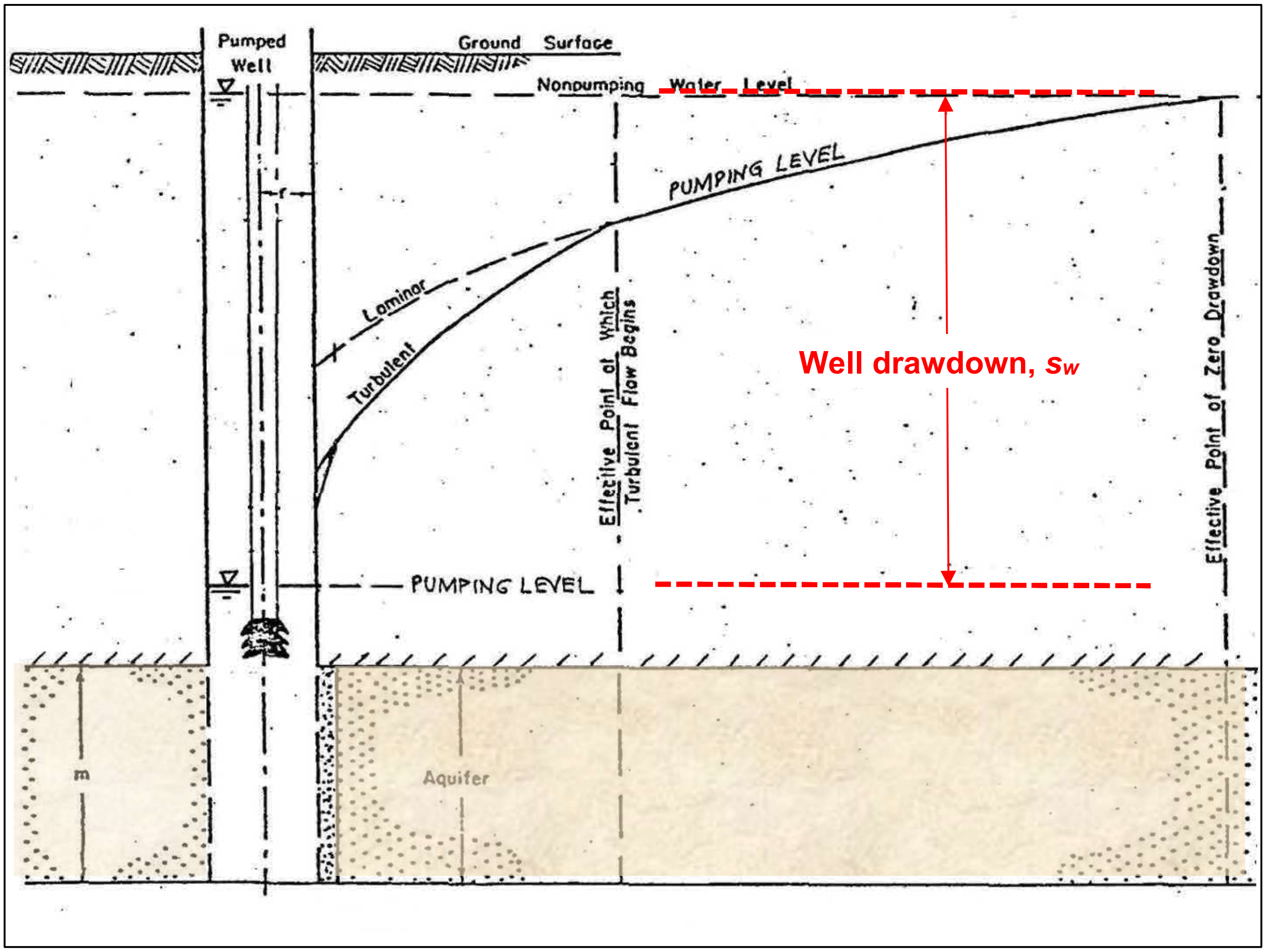


Figure 1. Drawdown in a pumping well
Adapted from Bruin and Hudson (1955)

[1] For simplicity, we will refer to "water level" rather than potentiometric level or hydraulic head.

2. Components of the drawdown in a pumping well

To support the interpretation of pumping well drawdowns it is important to first understand the components of the drawdown. Following the general approach of Walton (1962, 1970), the total drawdown is idealized as consisting of five components. Referring to Figure 2 and moving inwards from the formation to the inner casing, the head losses are:

1. s_a: the head loss due to *laminar* flow in the formation;
2. s_t: the additional head loss due to *turbulent* flow in the formation;
3. s_s: the additional head loss across a zone of reduced permeability around the well;
4. s_e: the additional head loss across the well screen; and
5. s_c: the additional head loss within the well casing itself.

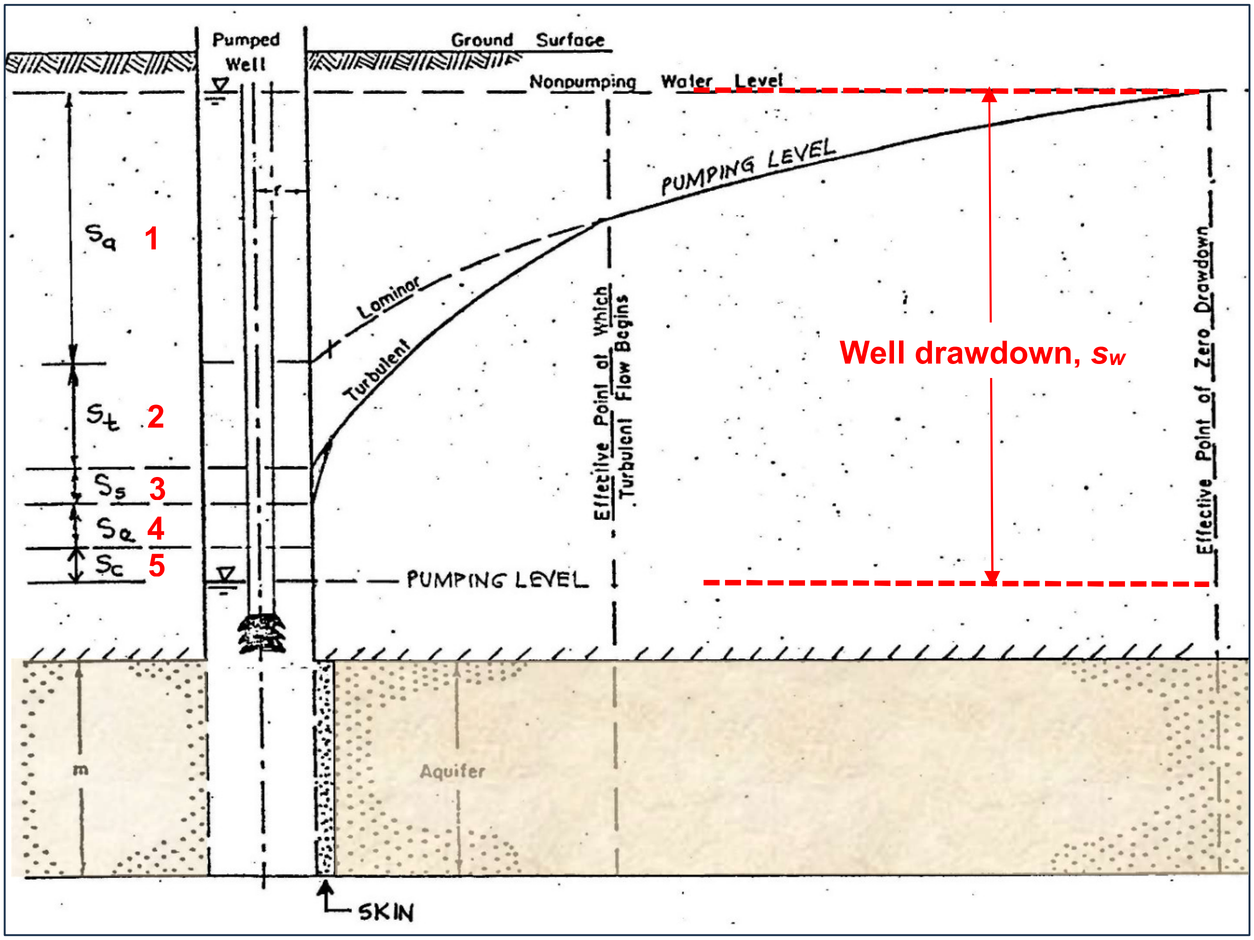


Figure 2. Components of drawdown in a pumping well

The components of the total drawdown in a pumping well are defined below.

1. s_a : Head loss due to *laminar* flow in the formation

The head losses due to laminar flow in the formation arise from friction losses as water is transmitted through the formation towards the pumping well. These head losses depend on the duration of pumping, the properties of the formation (transmissivity and storage coefficient), and the construction of the well (radius and extent of penetration).

2. s_t : Additional head loss due to *turbulent* flow in the formation

If the flow rate is sufficiently high, there may be additional head losses in the formation due to turbulent flow. If a well screened in a porous medium has been designed properly, there should be little possibility of turbulent flow in the formation. However, in fractured-rocks, pumping may induce velocities that are sufficiently high that flow is no longer laminar. Atkinson and others (1994) present an excellent treatment of turbulent flow in discrete fractures.

3. s_s: Additional head loss across a zone of reduced permeability around the well

Regardless of how carefully a well may be drilled, there is always the possibility that a zone of disturbed material may be created around it. The zone of disturbed material is usually referred to as a "skin", and the additional head losses due to its presence are referred to as a "skin effect". Skin effects may arise from the use of drilling mud in porous media, or by the closing off of fractures in rock. Skin effects may be mitigated to a certain extent by proper well development following drilling.

4. s_e: Additional head loss across the well screen

Head losses due to the flow of water across the well screen arise from the constriction in the flow as it passes through the openings of the well screen. These losses are generally referred to as entrance losses. If a well screen has been designed properly, these losses should not be significant. However, they may evolve through time if bacterial growth or mineral precipitates clog the well screen.

5. s_c: Additional head losses within the well itself

Additional head losses may occur within the well itself, due, for example, to turbulence arising from the constrictions around the pump appurtenances.

Total drawdown in the pumping well

The head losses in a pumping well are assumed to be additive. That is, the total drawdown in a pumping well is assumed to be the sum of drawdowns due to each of the components:

$$s_w(t) = s_a + s_t + s_s + s_e + s_c \quad (1)$$

Some researchers have attempted to quantify some or all of the five components of the drawdown indicated in Equation (1) (see for example Barker and Herbert, 1992a,b; and Atkinson et. al., 1994). However, on a practical level, it is generally not feasible to distinguish between all of them. A simplified model will be discussed in these notes, distinguishing only between the laminar head losses in the formation, additional head losses across a skin zone, and additional turbulent head losses:

$$s_w(t) = s_{formation} + \Delta s_{skin} + \Delta s_{turbulence} \quad (2)$$

The representation of these lumped components of the drawdowns are discussed in the following sections of the notes, starting with the representation of head losses in the formation.

3. Representation of head losses in the formation

The component of the drawdown due to head losses in the formation is shown schematically in Figure 3. The head losses in the formation are the difference in the groundwater level in the aquifer under non-pumping conditions and the level at the outside edge of the well screen or borehole at any subsequent time:

$$s_{formation}(t) = h(r_w)_{non-pumping} - h(r_w, t)_{pumping} \quad (3)$$

Here r_w is the effective radius of the pumping well and t is the elapsed time since the start of pumping. The effective radius is frequently assumed to be the outside radius of the borehole. The key aspect of formation losses is that, for either steady or transient flow, if flow in the formation is laminar, the change in the water level (drawdown) should be a linear function of the flow rate from the formation:

$$s_{formation}(t) = Q_{formation} \times F(r_w, t) \quad (4)$$

Here $F(r_w,t)$ denotes a particular aquifer model. When pumping starts the initial water withdrawn from the well is taken from the well casing (wellbore storage). Some time will be required until the flow rate from the formation ($Q_{formation}$) is equal to the pumping rate from the well (Q).

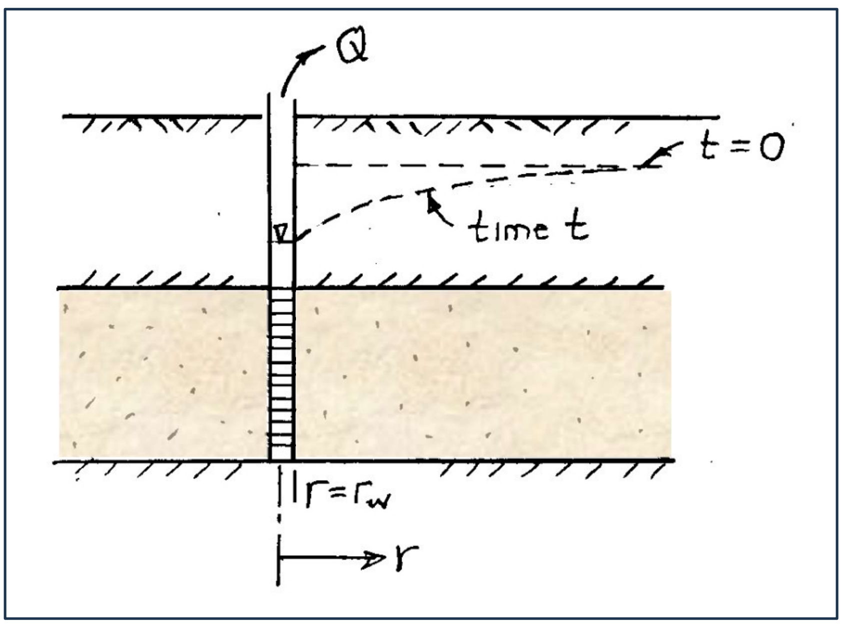


Figure 3. Idealization of the component of drawdown due to head losses in the formation

3.1 Head losses in the formation: Ideal steady conditions

The specific capacity of a well is defined as the ratio of the pumping rate and the drawdown in the well:

$$SC = \frac{Q}{s_w} \quad (5)$$

For steady flow to an ideal well, the specific capacity is constant. Referring to Figure 4, for this ideal case, the specific capacity is the same whether we consider a single point (Q/s_w) or the slope of the plot of pumping rate against drawdown, ($\Delta Q/\Delta s_w$).

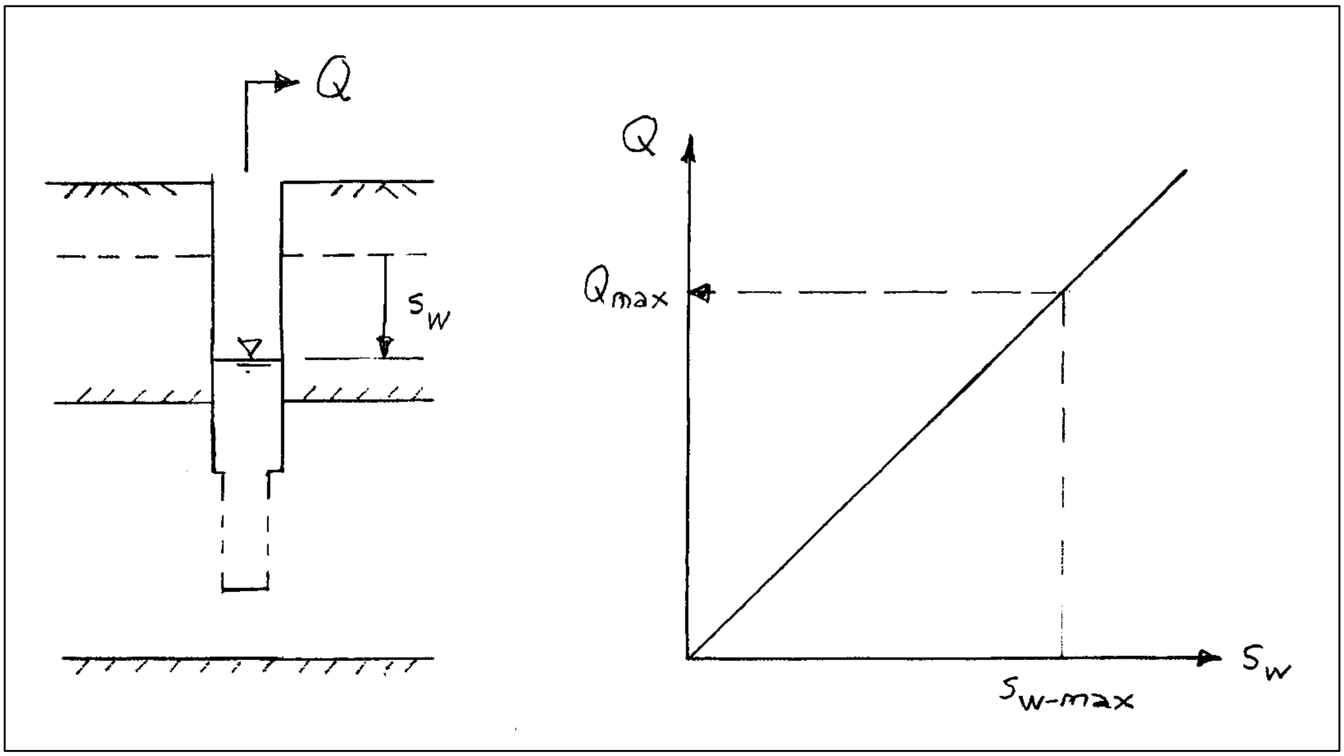


Figure 4. Estimation of the capacity of well with a constant specific capacity

Example 1

The results from a well performance test conducted on a municipal production well in Linnich (Ruhr Valley, Germany) are reproduced in Figure 5 (Langguth and Voigt, 1980). The top curve shows the pumping rate and the bottom curve shows the water level in the well. The data suggest that the water level in the well reaches a steady level during each pumping step.

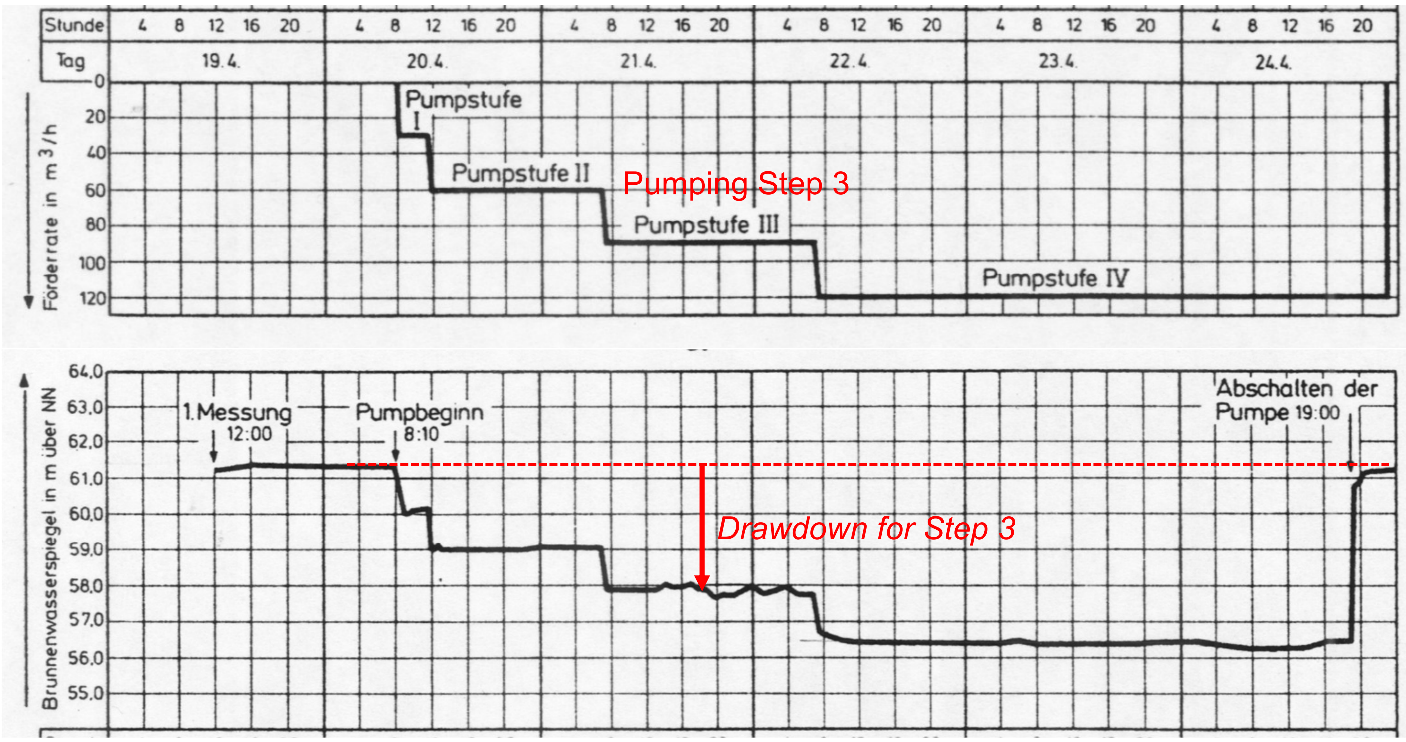


Figure 5. Data from municipal well testing, Linnich, Germany
(Data from Langguth and Voigt, 1980)

The pumping rates during the Linnich test are plotted against the stabilized drawdown in Figure 6. The results of the test approximate a straight line so we can estimate the specific capacity from either a single point or from the slope of the line of best fit through the data. For this example, the specific capacity is about 25 m^3/hr/m.

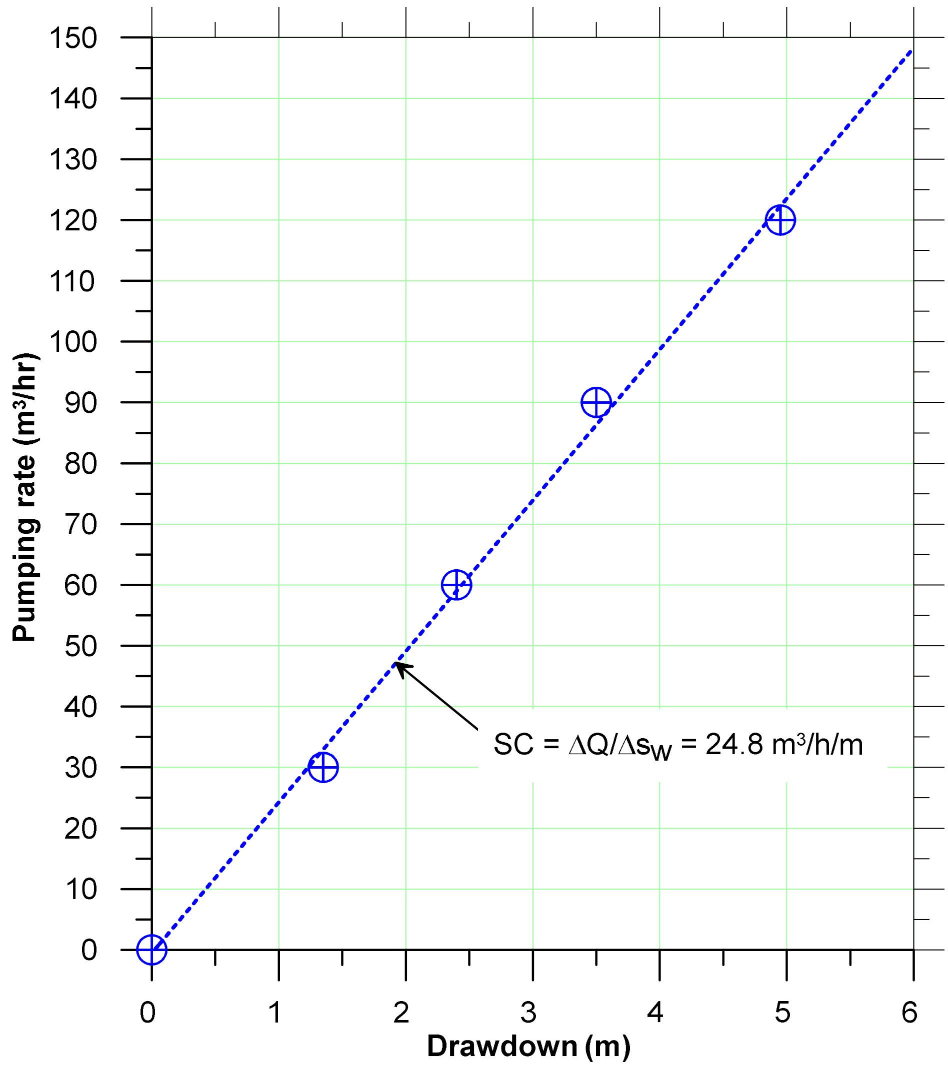


Figure 6. Reduced data from the Linnich step test

Example 2

The data in Figures 5 and 6 were obtained from a controlled test. Inferring the specific capacity from the operating record of a production well generally requires additional data processing. The data from a well in Aberfoyle, Ontario are shown in Figure 7. As shown in the figure, the daily pumping varied substantially, and the water level fluctuated on a daily basis.

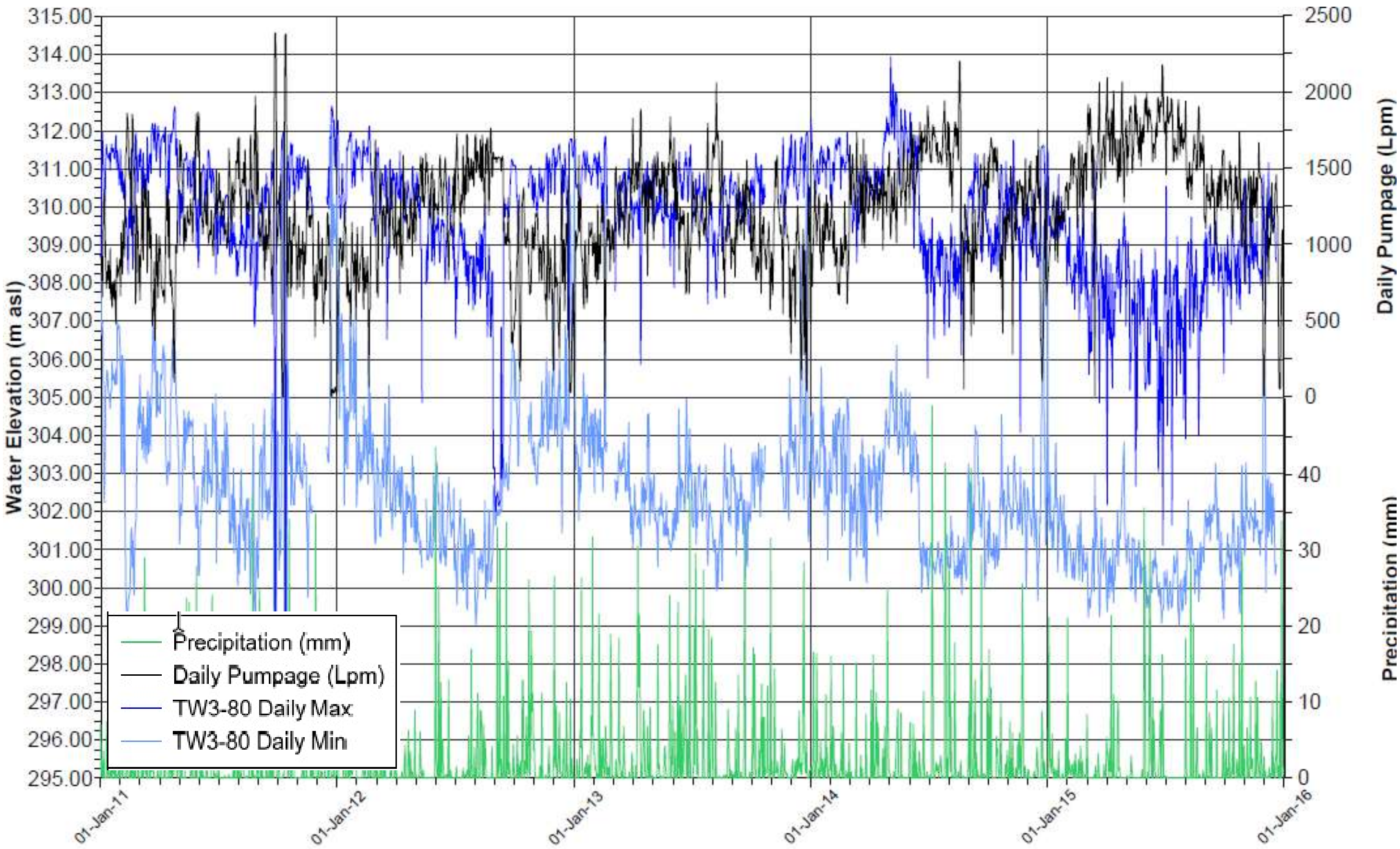
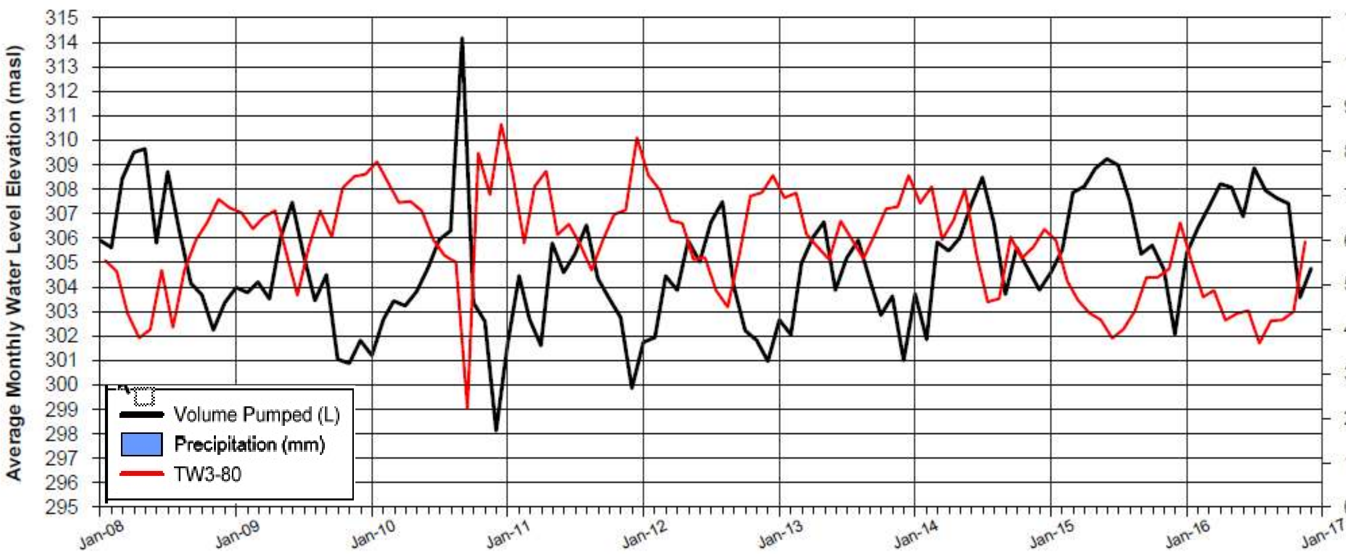


Figure 7. Performance record for an operating production well

The same data are plotted in Figure 8 but are averaged for each month of the record. In this form it is possible to estimate the long-term specific capacity of the well, about 160 L/min/m, and to assess whether the specific capacity changes through time.



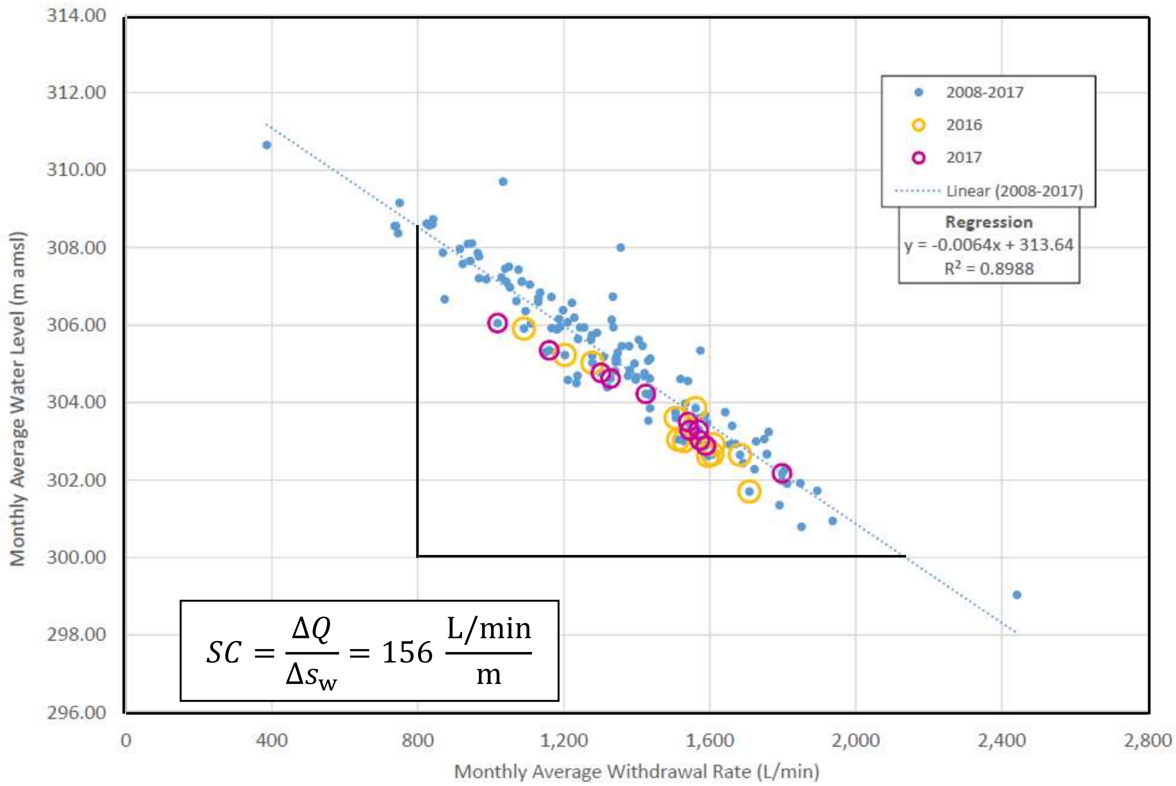


Figure 8. Monthly-average operating well performance data

If it is assumed that all of the drawdown in the pumping well is attributable to head losses due to laminar flow in the formation, the discharge to a well under steady conditions in an ideal confined aquifer is given by the Thiem solution:

$$Q = 2\pi K b \frac{(h_R - h_w)}{ln\{\frac{R}{r_w}\}} = \frac{2\pi T}{ln\{\frac{R}{r_w}\}} s_w \tag{6}$$

Here K is the horizontal hydraulic conductivity, b is the aquifer thickness, T is the transmissivity ($K \times b$), R is radius of influence, r_w is the radius of the pumping well, and h_R and h_w are the hydraulic heads at radial distances R and r_w from the center of the well.

For this case, the specific capacity is a constant given by:

$$SC = \frac{Q}{s_w} = \frac{2\pi T}{ln\{\frac{R}{r_w}\}} \tag{7}$$

We can only guesstimate the radius of influence R. However, since the ratio R/r_w appears in the logarithm term, the estimate of the specific capacity is relatively insensitive to the assumed value of R. As indicated on the table below, for variations of the assumed radius of influence by a factor of 100, the estimate of the specific capacity varies only by a factor of about 2.

R/r_w	$\frac{SC}{T}$
50	1.61
100	1.36
200	1.19
500	1.01
1000	0.91
2000	0.83
5000	0.74

We also note from the tabulated values that the ratio of the specific capacity and the transmissivity is relatively close to 1.0. The results of these simple calculations lead us to a rule of thumb.

> If most of the head losses in a well are due to laminar flow in the formation, as a first approximation the specific capacity of a well in a confined aquifer is equal to the transmissivity.

3.2 Head losses in the formation: Ideal transient conditions

For transient conditions, the Theis (1935) solution is invoked frequently to represent the component of the total drawdown due to head losses in the formation is given by:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} \times W(\frac{r_w^2 S}{4T}) \tag{8}$$

Here T denotes the transmissivity of the formation, S the storativity (confined storage coefficient) and t is the elapsed time. The term W denotes the Theis well function (the exponential integral). Application of the Theis solution assumes that the aquifer is extensive, uniform, isotropic, perfectly confined and pumped by a fully penetrating well. The drawdown due to laminar head losses in the formation are evaluated at the effective radius of the well.

As shown in Figure 9, for all but the earliest times after the start of pumping, drawdowns calculated with the Theis well function and Cooper and Jacob (1946) approximation are indistinguishable.

Replacing the Theis well function by the Cooper-Jacob approximation, the drawdown due to head losses in the formation are given by:

$$W(\frac{r_w^2 S}{4Tt}) \cong -0.5772 - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\} \tag{9}$$

Invoking this approximation, Equation (6) can be expanded as:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T}\left[ln\{EXP\{-0.5772\}\} - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right]$$

Making use of the properties of the log function:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} ln\left\{EXP\{-0.5772\}\frac{4Tt}{{r_w}^2 S}\right\}$$

$$= \frac{Q_{formation}}{4\pi T} ln\left\{\frac{2.246\ Tt}{{r_w}^2 S}\right\}$$

Changing to base 10 logarithms:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} 2.303\ log_{10}\left\{\frac{2.246\ Tt}{{r_w}^2 S}\right\} \tag{10}$$

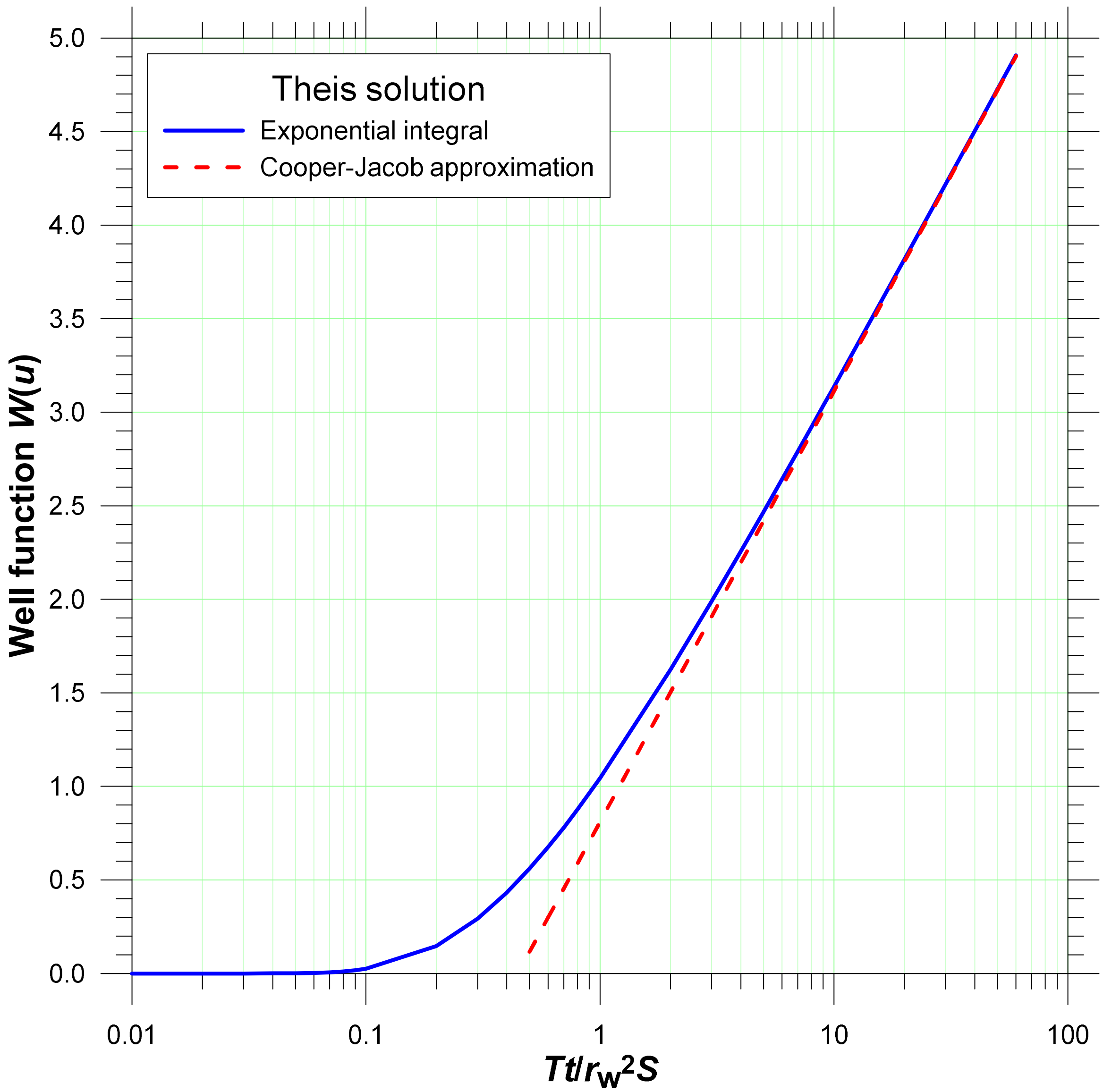


Figure 9. Theis well function and its approximation

4. Preliminary estimation of the transmissivity from the specific capacity of a pumping well

Transmissivity data are frequently limited in regional groundwater studies. Controlled pumping tests with observation wells are often available at only a few locations. However, the drilling logs for domestic supply wells contain information that can supplement the available data. In particular, these logs generally report pumping data that can be used to calculate specific capacities for the wells, and these specific capacities can be correlated to transmissivity with simple models. These correlations yield reconnaissance-level estimates of transmissivity. Where more detailed data are available, specific capacity values can also serve to provide simple check on the interpretations. Here we describe a simple approach for estimating the transmissivity from specific capacity data. The crucial assumption of the analysis is that the drawdowns in the pumping well are due primarily to head losses in the formation.

The specific capacity is defined as the ratio of the pumping rate (*Q*) and the drawdown in the pumping well (s_w):

$$SC = \frac{Q}{s_w} \tag{11}$$

If well losses and any effects of wellbore storage are neglected, the specific capacity can be estimated by evaluating the Theis solution at the radius of the wellbore, r_w:

$$SC = \frac{Q}{s_w} = \frac{4\pi T}{W\left(\frac{r_w^2 S}{4Tt}\right)} \tag{12}$$

The transmissivity can be back-calculated from the reported value of the specific capacity with known or assumed values for the well radius and storage coefficient:

$$T = \frac{1}{4\pi} W\left(\frac{r_w^2 S}{4Tt}\right) \times SC \tag{13}$$

Equation (13) is an implicit function of the transmissivity *T*. Although it is possible to estimate *T* using a root-finding algorithm, a simpler approach is illustrated here. For a particular well size and duration of pumping, it is possible to use Equation (13) directly to plot the relation between the *SC* and *T*. The transmissivity can then be estimated directly from the plot. We can develop our own plots for typical well diameters and durations of pumping.

The relation between specific capacity and transmissivity for typical conditions reported in water well records in Ontario is shown in Figure 10. The relationship is shown for a typical range of storage coefficients for confined conditions ($S = 1\times10^{-5}$ to 1×10^{-3}). The results plotted in Figure 10 demonstrate that the specific capacity is relatively insensitive to the value assumed for the storage coefficient.

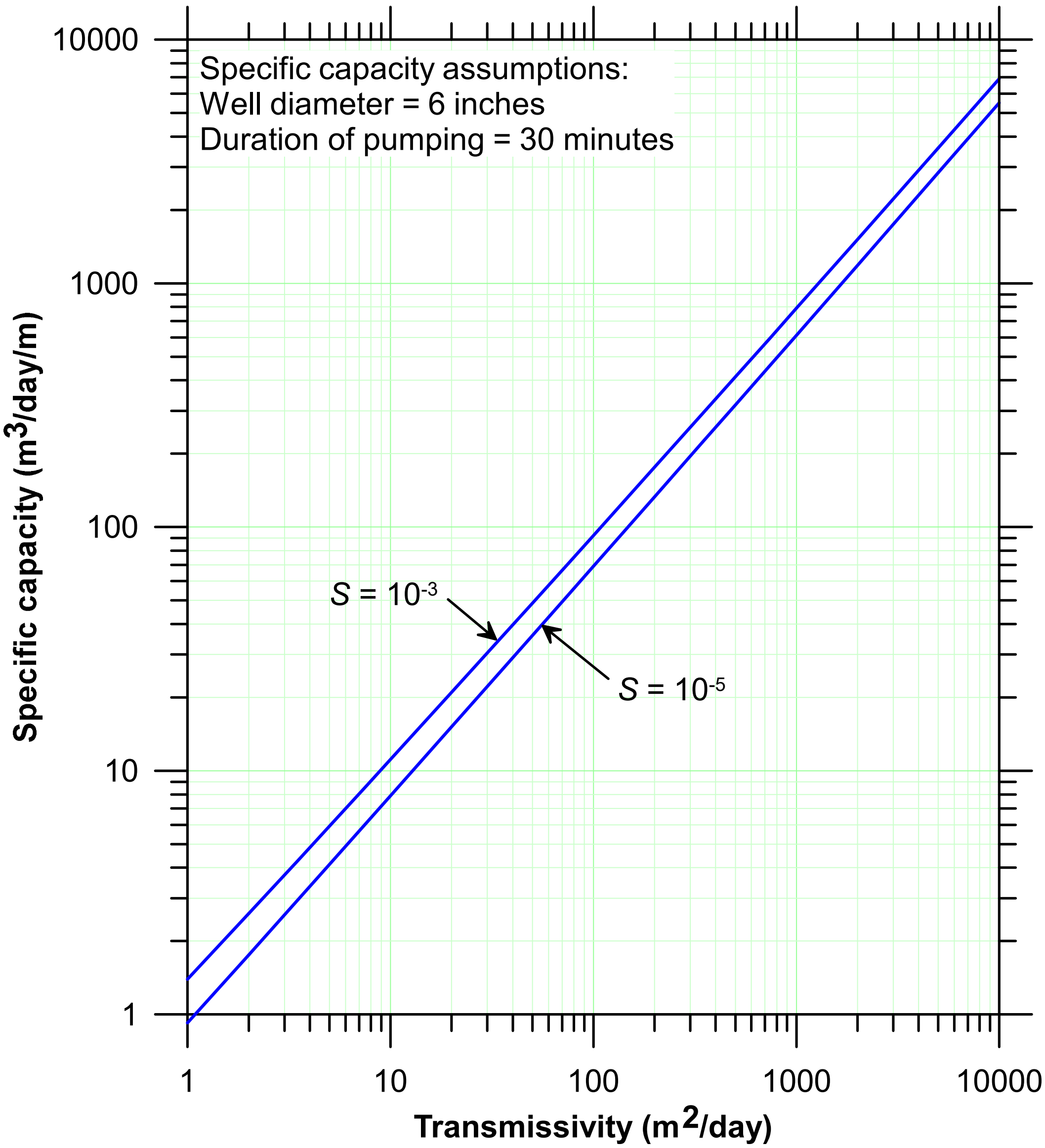


Figure 10. Specific capacity-transmissivity relation

The results shown in Figure 10 suggest that the log-transformed specific capacity is nearly a linear function of the log-transformed transmissivity range of 1 to 10,000 m^2/day. As shown in Figure 11, over this range the exact results are matched relatively closely with the simple relation (assuming consistent units):

$$T \approx 1.3 \times \ SC \qquad (14)$$

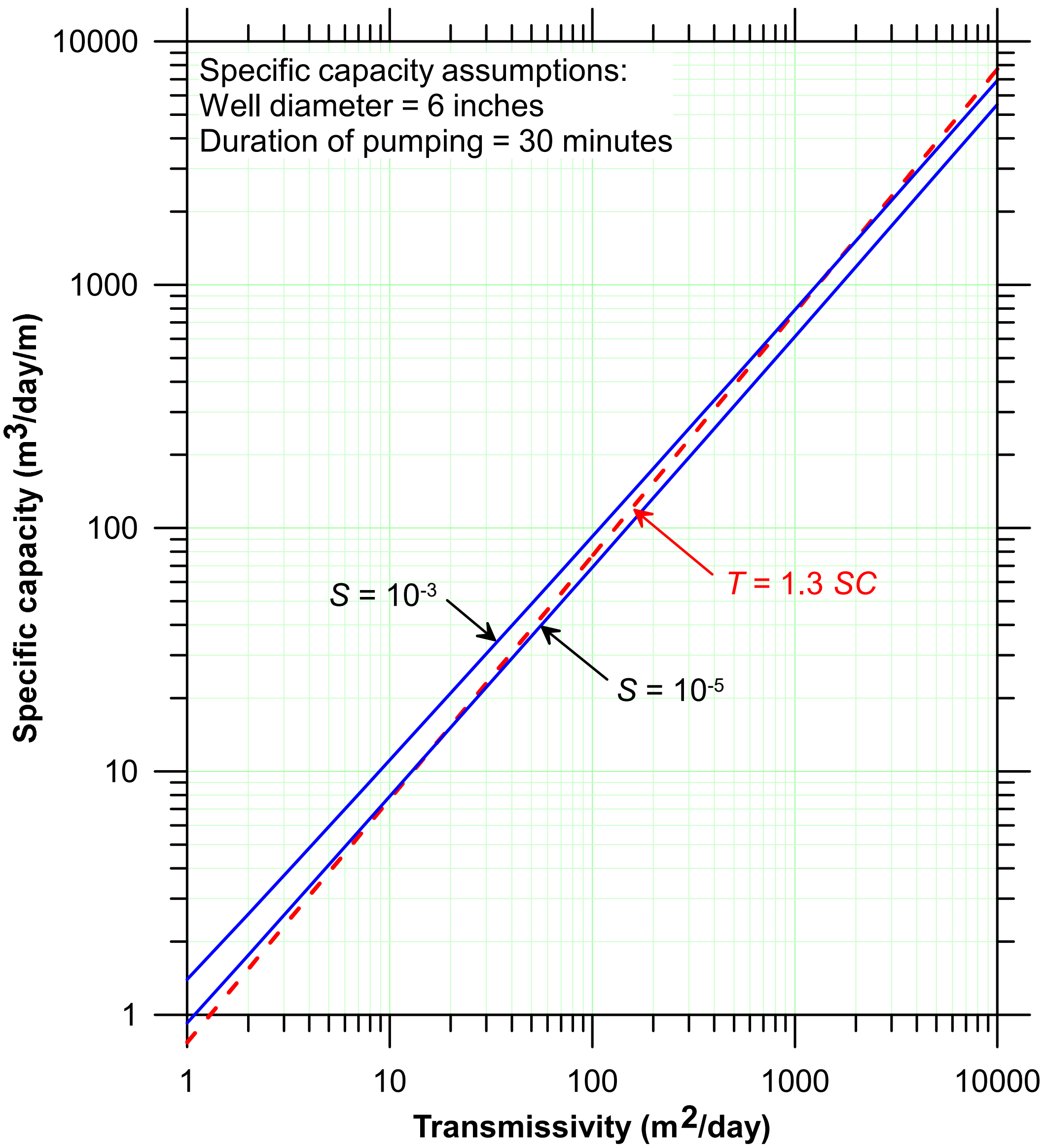


Figure 11. Specific capacity-transmissivity relation, with suggested correlation

If the specific capacity is specified in terms of U.S. gallons per minute (gpm) per foot of drawdown, and the transmissivity is reported in units of gallons/day-ft, the correlation becomes:

$$T \approx 1750 \times SC \quad (15)$$

The leading coefficient of 1750 is close to the value of 2000 presented in Driscoll (1986, p. 1021), which assumes the well is pumped for 1 day. The inferred correlation is superimposed on results plotted in Walton (1970, p. 317) in Figure 12 for a pumping period of 10 minutes. The results match closely, suggesting that the inferred correlation is appropriate for the shorter pumping periods typically reported in the water well records.

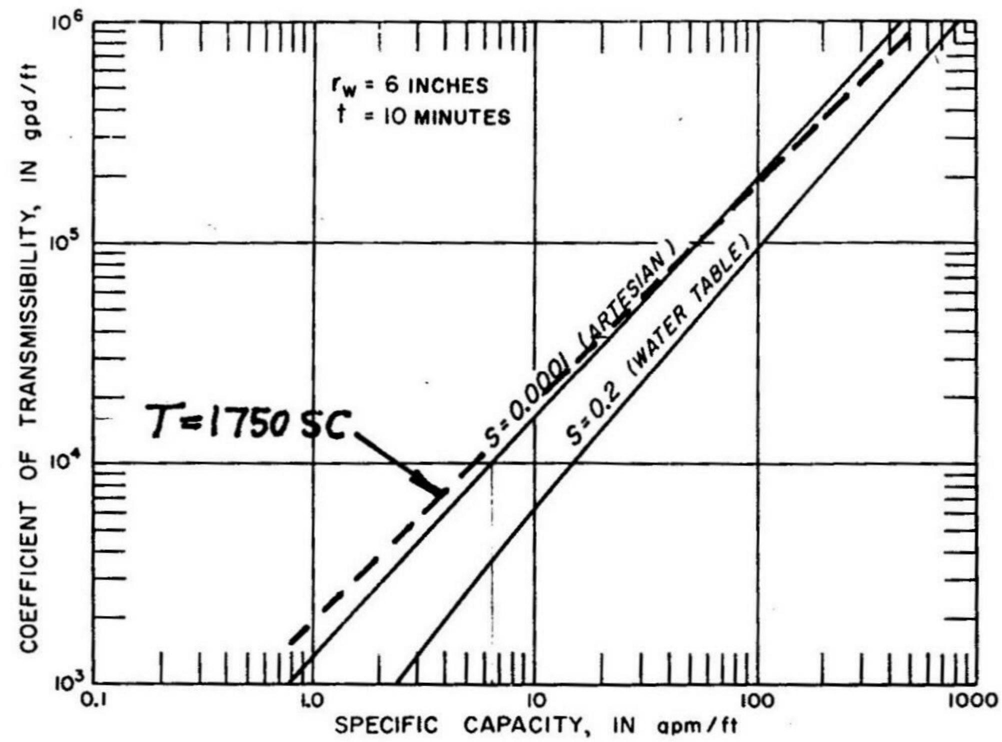


Figure 12. Specific capacity-transmissivity relation for brief pumping

The specific capacity is a weak function of the well radius and duration of pumping. The results of additional calculations suggest that Equations (14) and (15) do not need to be modified significantly to accommodate different well sizes or durations of pumping.

Case study: Rosemont, Ontario well PW3

The first-cut estimation of transmissivity from the specific capacity is also useful for conducting a quick check on more complete analyses. The data from a pumping test conducted at Rosemont, Ontario is used to illustrate the approach. Well PW3 was pumped for three days at an average rate of 0.6 L/s (51.84 m^3/d). The complete record of drawdowns is shown in Figures 13. The drawdown at the end of 60 minutes of pumping is 5.94 m. Therefore, the specific capacity after 60 minutes is:

$$SC = \frac{(51.84\ \mathrm{m^3/d})}{(5.94\ \mathrm{m})} = 8.73\ \mathrm{m^3/d/m}$$

The transmissivity estimated from specific capacity is:

$$T \approx 1.30 \times (8.73\ \mathrm{m^3/d/m}) = \mathbf{11.3\ m^2/d}$$

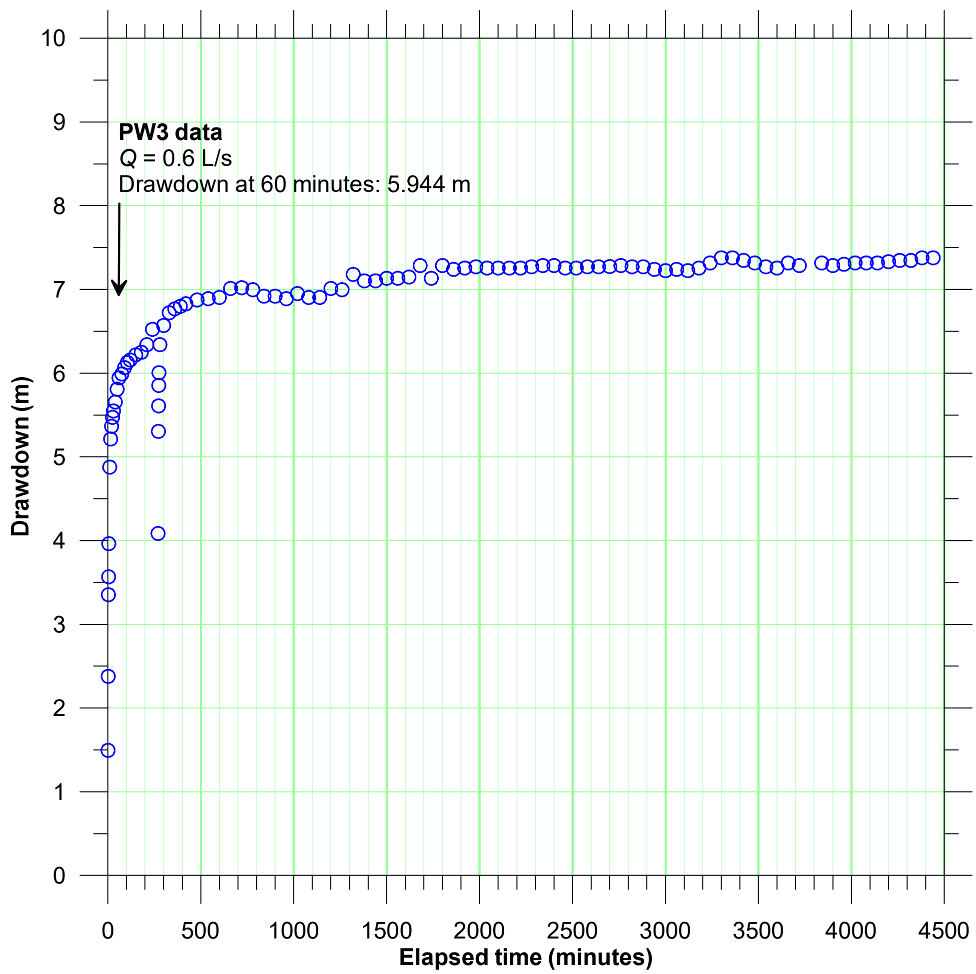


Figure 13. Drawdown record for the PW3 pumping test

The results of more rigorous analyses are shown in Figure 14. The transmissivity is estimated with the Cooper-Jacob analysis and with a match to the complete drawdown record with the Papadopulos and Cooper (1967) solution. A transmissivity of about **11 m²/day** is estimated from both analyses. The close agreement between the two analyses suggests that well losses do not have a significant influence on the estimation of transmissivity for this test.

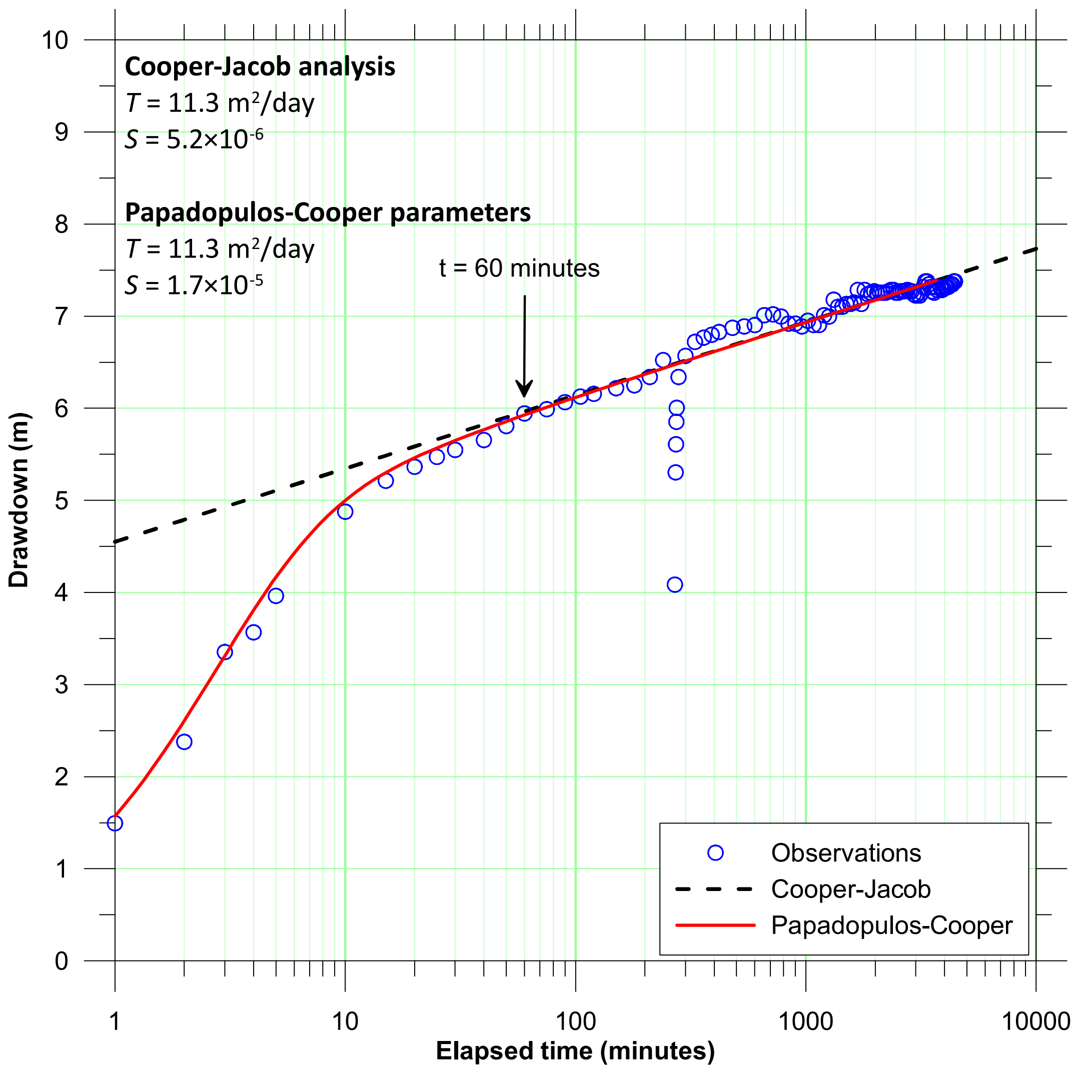


Figure 14. Rigorous analyses of the PW3 pumping test

The transmissivity estimated from the specific capacity for the Roseville well is close to the estimates developed from the more rigorous analyses of the complete drawdown record. This is not simply fortuitous. The availability of a complete drawdown record allows us the opportunity to confirm the following *in this case*.

- The time corresponding to the drawdown specified in the calculation of the specific capacity was sufficiently long for the effects of wellbore to dissipate. Referring to Figure 14, after about 30 minutes of pumping the differences between the Papadopulos and Cooper (1967) solution and the Cooper-Jacob straight line approximation are relatively small. This suggests that the effects of wellbore storage are almost completely dissipated within about 30 minutes so that the drawdown measured at 60 minutes provides a representative impression of the response of the formation.

- The early-time drawdowns are relatively small, which suggests that additional well losses are not significant. Therefore, the observed drawdowns in the pumping well provide a reliable impression of the head losses in the formation in the vicinity of the pumping well.

- The storage coefficients estimated from the Papadopulos-Cooper and Cooper-Jacob analyses are within the range of realistic storage coefficients for a confined aquifer that is relatively thin. This is consistent with the inference that the primary component of the observed drawdowns is head losses within the formation.

5. Representation of the additional head losses across a skin zone

Drilling and installing a pumping well generally cause some alteration in the properties of the formation around the wellbore. The zone of altered material is referred to as the *skin*. If the hydraulic conductivity of the skin is reduced relative to the formation, there will be additional head losses across the skin as shown schematically in Figure 15. The distance from the center of the well to the outer edge of the skin is designated r_s.

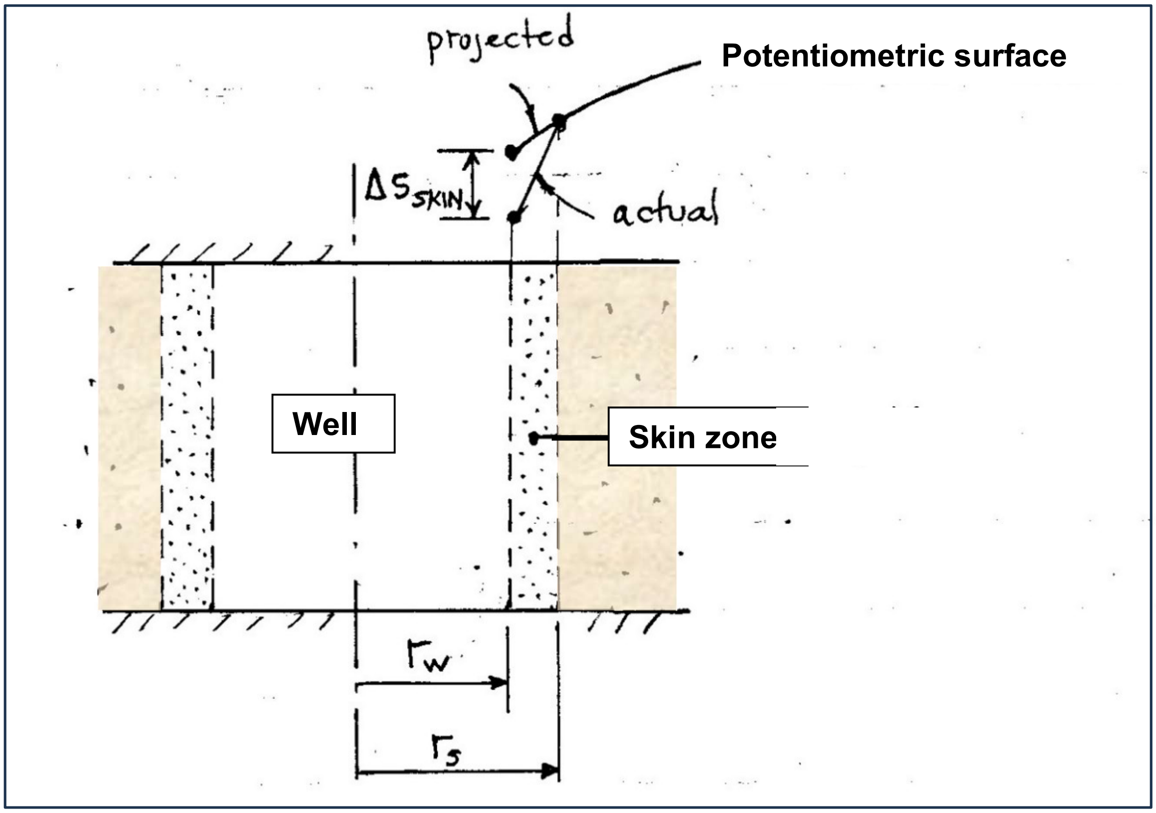


Figure 15. Schematic cross-section of a well surrounded by a skin zone

There are two key aspects of skin losses:

- Skin losses are established relatively quickly after pumping starts; and
- Skin losses are proportional to the pumping rate.

Ramey (1982) proposed that the effects of a zone of damaged material around the pumping well could be represented by a constant additional drawdown:

$$\Delta s_{skin} = \frac{Q}{4\pi T} 2S_w \tag{16}$$

Here S_w is referred to as the *dimensionless skin factor*.

Assuming that there is no storage within the skin zone it is possible to derive an analytical expression for the dimensionless skin factor (Hawkins, 1956):

$$S_w = \left(\frac{T - T_S}{T_S}\right) ln\left\{\frac{r_s}{r_w}\right\} \tag{17}$$

Here T and T_S are the transmissivities of the formation and the skin, respectively, and r_s is the radius of the skin. Equation (17) can be rearranged to read:

$$S_w = \left(\frac{T}{T_S} - 1\right)\ ln\left\{\frac{r_s}{r_w}\right\} \tag{18}$$

This definition is presented as Eq. 2.10 in the classic petroleum engineering text of Earlougher (1977). In this form it is clear why petroleum engineers use the terminology “positive skin” to denote the effect of a reduced permeability of the skin, and “negative skin” to denote the effect of an increased permeability of the skin relative to the formation. In practice, we cannot estimate the extent of the skin zone or isolate its properties. Therefore, S_w is generally treated as a lumped parameter. As will be shown in a subsequent section of these notes, the presence of a skin can frequently be inferred from the estimation of a non-physical storage coefficient.

6. Representation of the additional head losses due to partial penetration

Additional head losses occur when a pumping well does not penetrate the full thickness of an aquifer. The conceptual model of a partially penetrating well is shown in Figure 1.

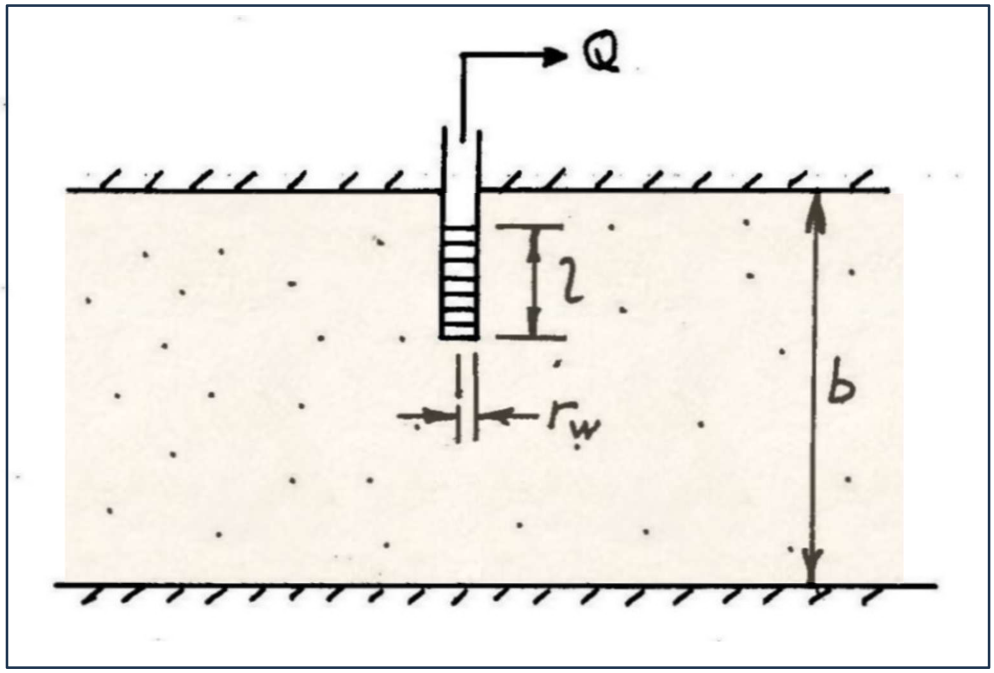


Figure 16. Conceptual model for a partially penetrating well

Rigorous analyses of flow to partially penetrating pumping wells suggest that the additional head losses caused by partial penetration are established relatively quickly and are directly proportional to the pumping rate (Hantush, 1961). Therefore, they have the same general form as skin losses. The losses due to partial penetration are written in terms of a *pseudo-skin coefficient, S_{pp}*:

$$\Delta s_{pp} = \frac{Q}{4\pi T} 2S_{pp} \qquad (19)$$

Several approaches have been developed to estimate the additional head losses due to partial penetration. Brons and Marting (1961) developed a simple approach that in our experience closely approximates results obtained with more elaborate calculations:

$$S_{pp} = \left(\frac{b-l}{l}\right)\left[ln\left\{\frac{b}{r_w}\right\} - G\left(\frac{l}{b}\right)\right] \qquad (20)$$

Here *b* is the aquifer thickness, *l* is the length of the well screen, and $G\left(\frac{l}{b}\right)$is a function tabulated in Brons and Marting (1961).

Bradbury and Rothschild (1985) used regression to develop the following functional form from the tabulated values of G:

$$G\left(\frac{l}{b}\right) \cong 2.948 - 7.363\left(\frac{l}{b}\right) + 11.447\left(\frac{l}{b}\right)^2 - 4.675\left(\frac{l}{b}\right)^3 \tag{21}$$

The values tabulated by Brons and Marting (1961) are plotted in Figure along with the regression of Bradbury and Rothschild (1985). As shown in the figure, the results obtained with the regression relation match closely the values in Brons and Marting (1961).

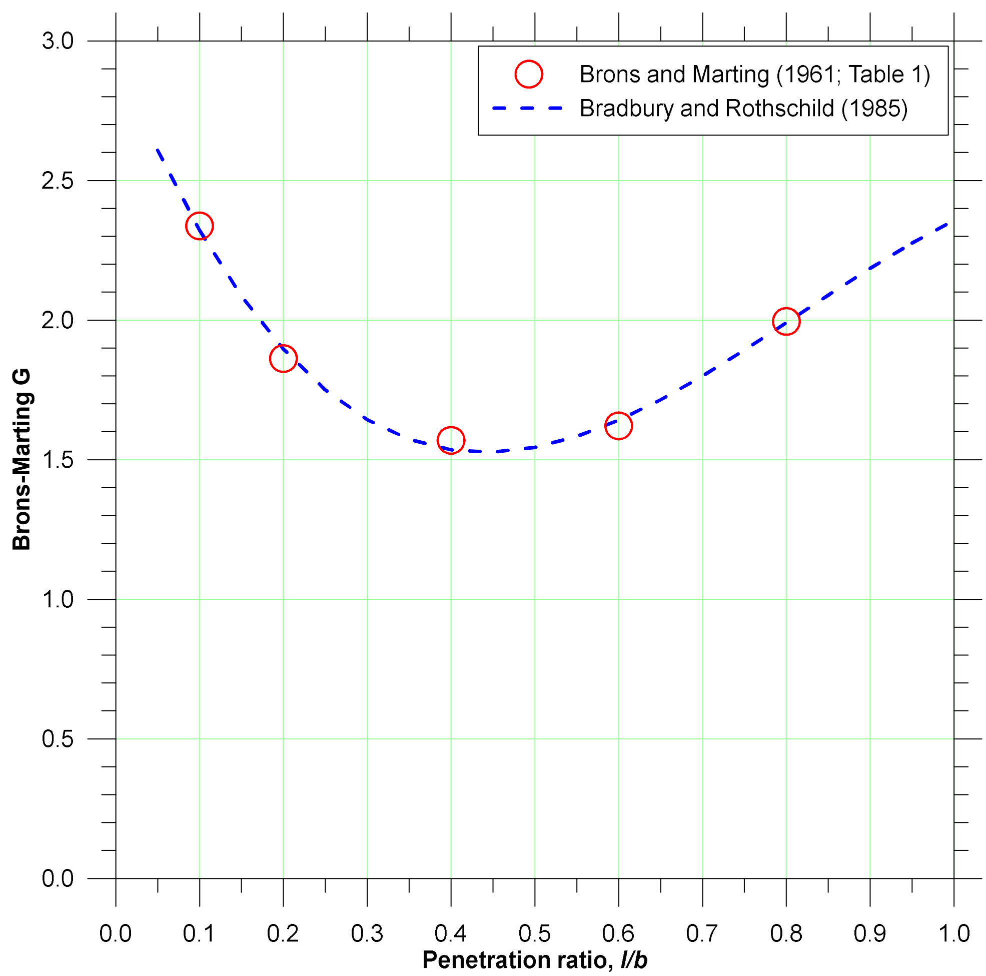


Figure 17. Values of the Brons-Marting function *G* for partially penetrating wells

7. Representation of the additional head losses due to turbulent flow near or within the well

The velocity of water in the immediate vicinity of the well may be sufficiently high that flow is turbulent. Flow may also be turbulent within the well casing itself and around the appurtenances may be relatively high, and flow may be turbulent. Jacob (1946) proposed a simple phenomenological approach for estimating the head losses due to turbulence in the well itself. There are two key aspects of the Jacob model of in-well turbulent losses:

- Turbulent losses are established relatively quickly after pumping starts; and
- Turbulent losses are proportional to the pumping rate squared.

The Jacob model is expressed as:

$$\Delta s_{turbulence} = CQ^2 \tag{22}$$

The parameter C is designated the *well loss coefficient*. Rorabaugh (1953) suggested that Jacob's model was not always appropriate, and proposed the following generalization:

$$\Delta s_{turbulence} = CQ^P \tag{23}$$

Here P is designated the *well loss exponent*. Rorabaugh reported exponents that were not too different from 2.0, and we recommend using Jacob's model except where there is compelling evidence that P should not be 2.0. The well loss coefficient C has the units of [drawdown]/[Pumping rate]^P. If the pumping rate is reported in m^3/day the units of drawdown are m, and it is assumed that $P = 2$, then C has units of $m/(m^3/day)^2$, which is equivalent to day^2/m^5.

The well loss coefficient C is a fitting parameter. The most reliable estimates of C are derived from the results of step tests, as discussed later in these notes. In the absence of site-specific data, we suggest that the general guidance provided by Walton (1962; p. 27) be used to assess preliminary values.

Suggested value of the nonlinear well loss coefficient, *C*

Condition of well	C (sec^2/ft^5)	C (day^2/ft^5)	C (sec^2/m^5)
Properly designed and developed	$C < 5$	$C < 6.7\times10^{-10}$	$C < 1900$
Mild deterioration	$C < 10$	$C < 1.3\times10^{-9}$	$C < 3800$
Well beyond rehabilitation	$C > 10$	$C > 1.3\times10^{-9}$	$C > 3800$

8. Complete transient analysis with the incorporation of additional well losses

Drawdown data from typical tests confirm that the additional drawdowns due to skin effects and in-well losses are established relatively soon after pumping starts, compared with the head losses in the formation. If it is assumed that the Theis conceptual model is applicable, substituting the expressions introduced previously for Δs_{skin} and $\Delta s_{turbulence}$ in Equation (2) yields the following expression for the evolution of the total drawdown in a pumping well:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303 \log \left\{2.246\, \frac{Tt}{r_w^2 S}\right\} + \frac{Q}{4\pi}\, 2S_w + CQ^2 \tag{24}$$

Here it is assumed that effects of wellbore storage have dissipated. When evaluated at small values of the radial distance r, the Cooper-Jacob approximation is appropriate for all but the earliest values of time.

Expanding the log term:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303 \left(\log \left\{2.246\, \frac{T}{r_w^2 S}\right\} + \log\{t\}\right) + \frac{Q}{4\pi T}\, 2S_w + CQ^2$$

Rearranging:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303\ \log\{t\} + \frac{Q}{4\pi}\, 2.303 \log \left\{2.246\, \frac{T}{r_w^2 S}\right\} + \frac{Q}{4\pi T}\, 2S_w + CQ^2 \tag{25}$$

The first term is a function of time, but the other three terms are constant. In other words, the time rate of change of drawdown is not affected by the processes that cause additional head losses in the pumping well. Since the Cooper-Jacob analysis is based on the rate of change of drawdown rather than the absolute magnitude of the drawdown, it is possible to obtain a reliable estimate of the transmissivity from a Cooper-Jacob straight-line analysis, regardless of the magnitudes of the skin losses and turbulent well losses.

Example calculations:

Let us consider an ideal aquifer that is homogeneous, horizontal, perfectly confined, infinite in extent, and pumped by a fully penetrating well. The aquifer is assumed to have a transmissivity of 8.64 m^2/day and a storativity of 1.0×10^{-4}. These properties are typical of a medium sand aquifer that is 10 m thick. The aquifer is pumped at a constant rate of 104.54 m^3/day, and the pumping well has a radius of 0.05 m. Let us further assume that the pumping well losses are characterized by the following parameters:

- $S_w = 0.5193$; and
- $C = 1.340\times10^{-4}$ $m^{-5}d^2$.

These values have been specified only for illustrative purposes – under no circumstances would we report estimates from a real test with so many significant figures.

The total drawdowns in the pumping well are plotted in Figure 18. The dashed line in Figure 18 represents the drawdowns in the pumped well that are due only to head losses in the formation. The drawdown axis is arithmetic, and therefore the additional head losses in the pumping well appear as a constant offset. Both drawdown curves have the same slope on a Cooper-Jacob semilog plot.

The slope of the line plotted through the drawdown data is approximately 2.2 m. Therefore, the transmissivity is estimated as:

$$T = 2.303 \frac{Q}{4\pi} \frac{1}{SLOPE}$$

$$= 2.303 \frac{(104.54 \text{ m}^3/\text{d})}{4\pi} \frac{1}{(2.2 \text{ m})} = \mathbf{8.7\ m^2/d}$$

The estimated transmissivity is close to the specified value of 8.64 m^2/d.

The transmissivity with the Cooper-Jacob analysis is estimated from the semilog slope of the drawdowns and *not* the magnitudes of the drawdowns. Therefore, for a constant-rate pumping test the transmissivity estimate is not affected by the constant offsets of the additional well losses.

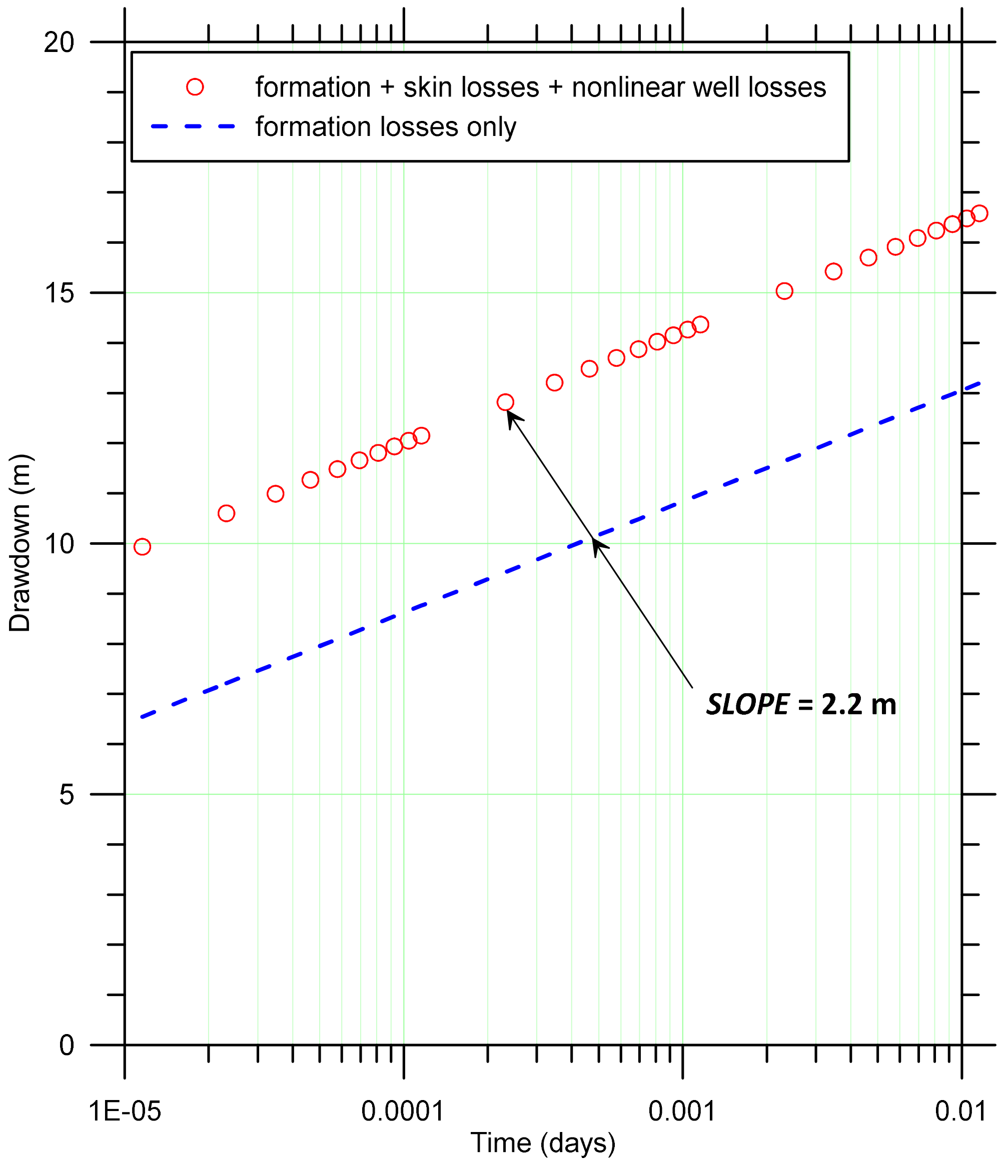


Figure 18. Drawdowns at the pumping well

9. Diagnosis of additional well losses

In the context of the analysis of the drawdowns in a pumping well, the estimation of a non-physical value of the storage coefficient is frequently a good indicator of the presence of additional well losses.

The storage coefficient is estimated with the Cooper-Jacob analysis according to:

$$S = 2.2459\frac{Tt_0}{r_w^2}$$

Here t_0 is the intercept of the straight-line approximation. Referring to the expanded version of Figure 18 shown in Figure 19, the value of t_0 is about 4×10^{-10} days. Therefore, the fitted storage coefficient is estimated as:

$$S = 2.2459\frac{(8.64\ \mathrm{m}^2/\mathrm{d})(4 \times 10^{-1}\ \ \text{days})}{(0.1\ \mathrm{m})^2} = \mathbf{7.7 \times 10^{-7}}$$

The "fitted" storage coefficient is more than a factor 100 less than the specified value and is well outside of the range of typical values of the storage coefficient for confined sand and gravel aquifers, from about 10^{-5} to 10^{-4}.

The storage coefficient is estimated from the intercept of the plot; therefore, in contrast to the estimation of the transmissivity, the inferred magnitude of the storage coefficient *does* depend on the magnitudes of the drawdowns. When we use the Cooper-Jacob straight-line analysis, we effectively estimate a storativity that accounts in a "lumped" sense for the effects of storage and additional well losses. Although we do not obtain a true estimate of the storage coefficient, its estimation still has useful diagnostic value. Estimation of an unrealistic value of the storage coefficient suggests there are additional sources of drawdown beyond head losses in the formation.

If all we have are the data from the pumping well when it is pumped at a constant rate, then we must accept the fact that we cannot obtain reliable estimates of the storativity and the well loss parameters. The data are not sufficient to characterize the performance of the pumping well.

We use this example to illustrate another subtle point. In Figure the drawdowns in the pumping well are re-plotted on log-log axes in anticipation of matching the observations with the Theis solution. In contrast to the Cooper-Jacob analysis, the additional head losses in the pumping well do not appear as a constant offset. They are in fact difficult to detect on a log-log plot. Furthermore, the estimate of transmissivity will be affected by the additional well losses.

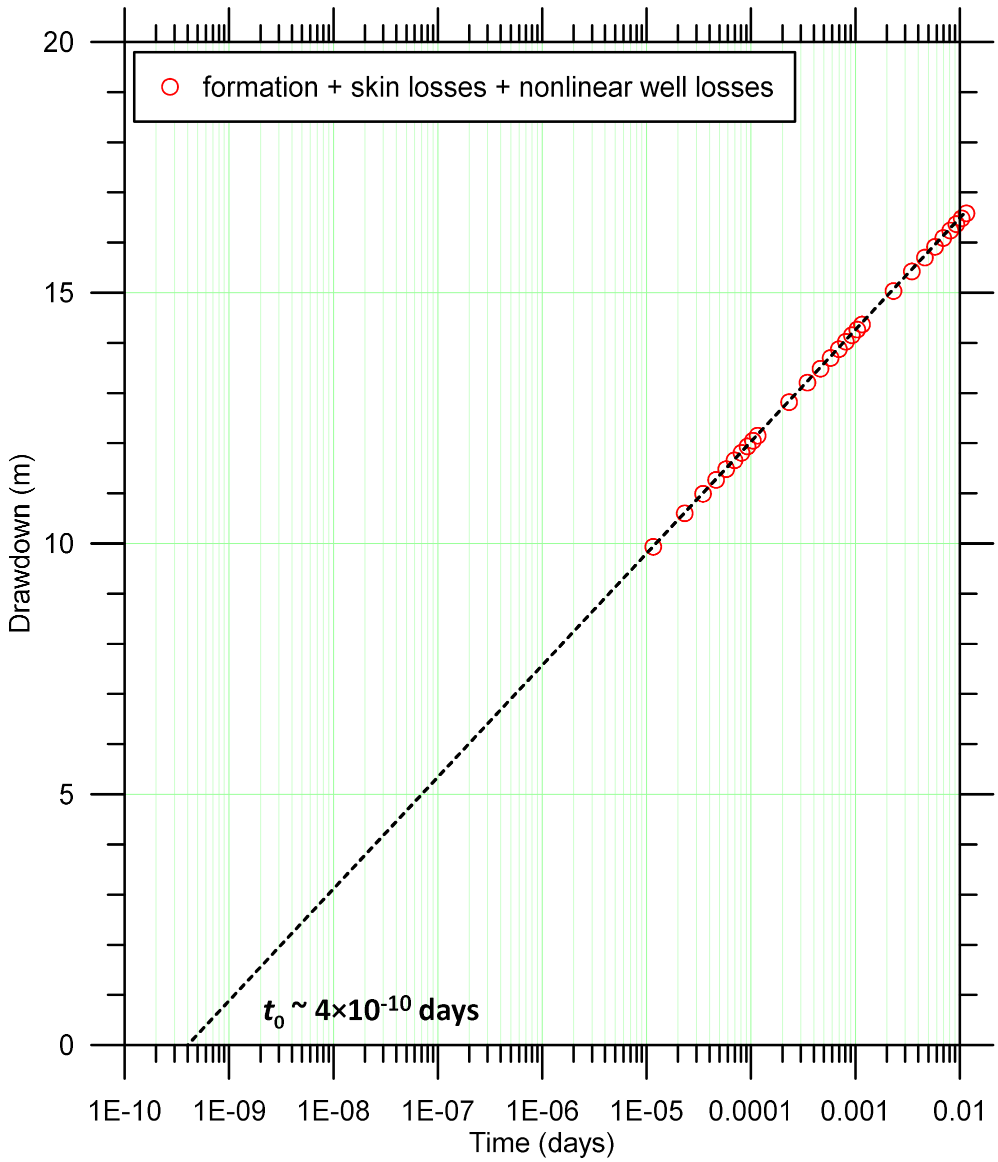


Figure 19. Drawdowns at the pumping well, with expanded time axis

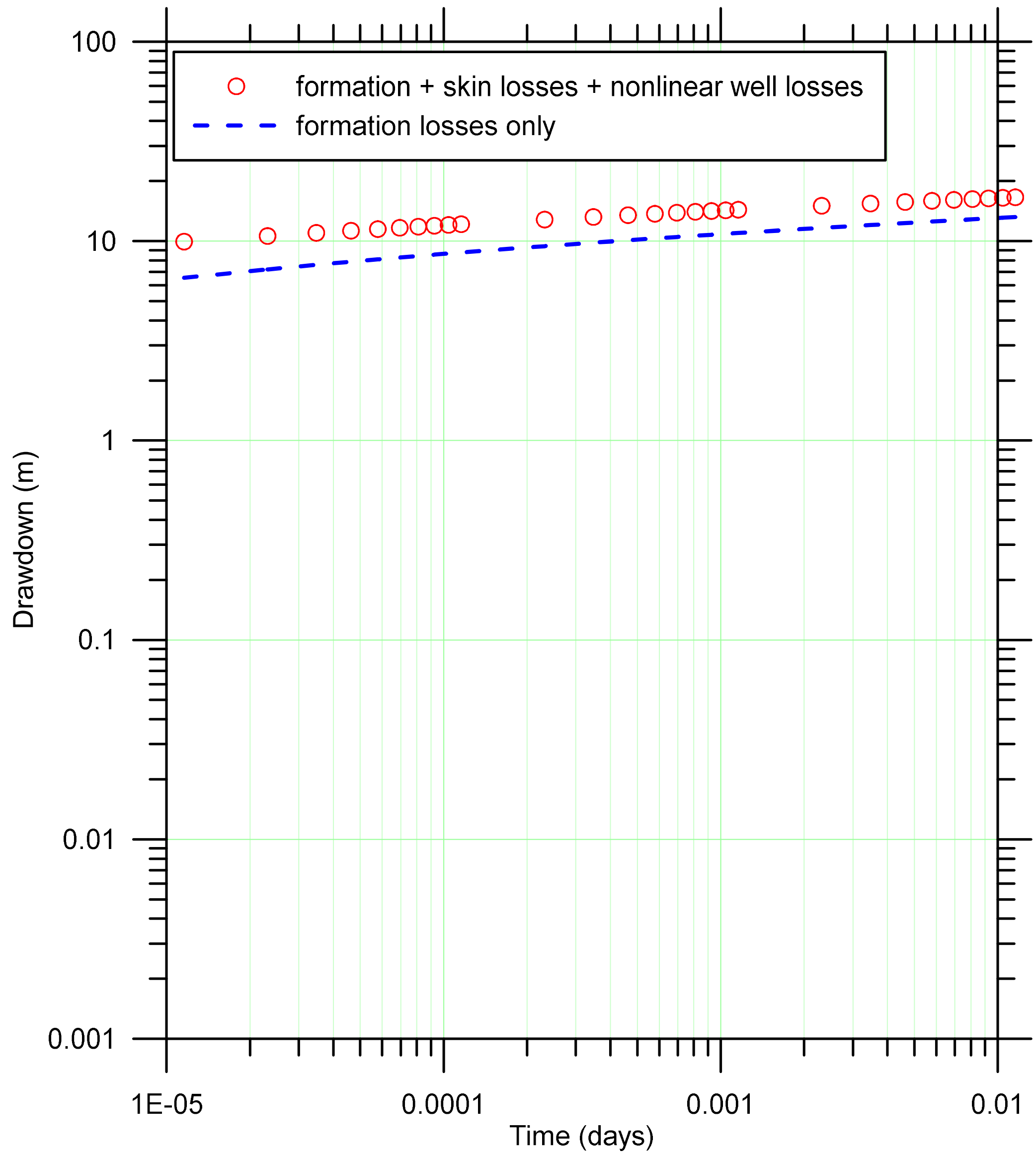


Figure 20. Drawdowns at the pumping well, log-log axes

10. Further investigation of additional well losses

As indicated in the previous section, the estimation of a non-physical value of the storage coefficient from the pumping well drawdowns during a constant-rate test is frequently a good indicator of the presence of additional well losses. This is explored further for a simple example of a well surrounded by a skin zone.

Assuming the drawdowns in a pumping well are attributed only to laminar head losses in the formation and head losses across the skin, the drawdown in the well is given by:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w{}^2 S}{4Tt}\right\}\right] + \frac{Q}{4\pi} 2S_w \quad (26)$$

Equation (23) can be expanded as:

$$s_w(t) = \frac{Q}{4\pi T}\left[ln\{EXP\{-0.5772\}\} - ln\left\{\frac{r_w{}^2 S}{4Tt}\right\} + ln\{EXP\{2S_w\}\}\right]$$

Making use of the properties of the log function:

$$s_w(t) = \frac{Q}{4\pi T} ln\left\{EXP\{-0.5772\}\frac{4Tt}{r_w{}^2 S}\ EXP\{2S_w\}\right\}$$

$$= \frac{Q}{4\pi T} ln\left\{\frac{2.246\,Tt}{r_w{}^2 S\ EXP\{-2S_w\}}\right\}$$

Changing to base 10 logarithms:

$$s_w(t) = \frac{Q}{4\pi} 2.303\ log_{10}\left\{\frac{2.246\,Tt}{r_w{}^2 S\ EXP\{-2S_w\}}\right\} \quad (27)$$

In this form we see that the pumping well drawdowns correspond to those that would be matched with the Cooper-Jacob approximation with an "effective" storage coefficient given by:

$$S_{\text{eff}} = S\ EXP\{-2S_w\} \quad (28)$$

The results of example calculations are shown in Figure 21. For these calculations a typical value of the storage coefficient for a confined sand and gravel aquifer is assumed, $S = 1.0\times10^{-4}$. As shown in the figure, when the pumping well is surrounded by a zone of reduced hydraulic conductivity (positive skin), an unrealistically small value of the storage coefficient is inferred from a Cooper-Jacob analysis ($S_{eff} \sim 2.0\times10^{-6}$). In contrast, when the pumping well is surrounded by a zone of increased hydraulic conductivity (negative skin), an unrealistically large value of the storage coefficient is inferred from a Cooper-Jacob analysis ($S_{eff} \sim 5.0\times10^{-3}$).

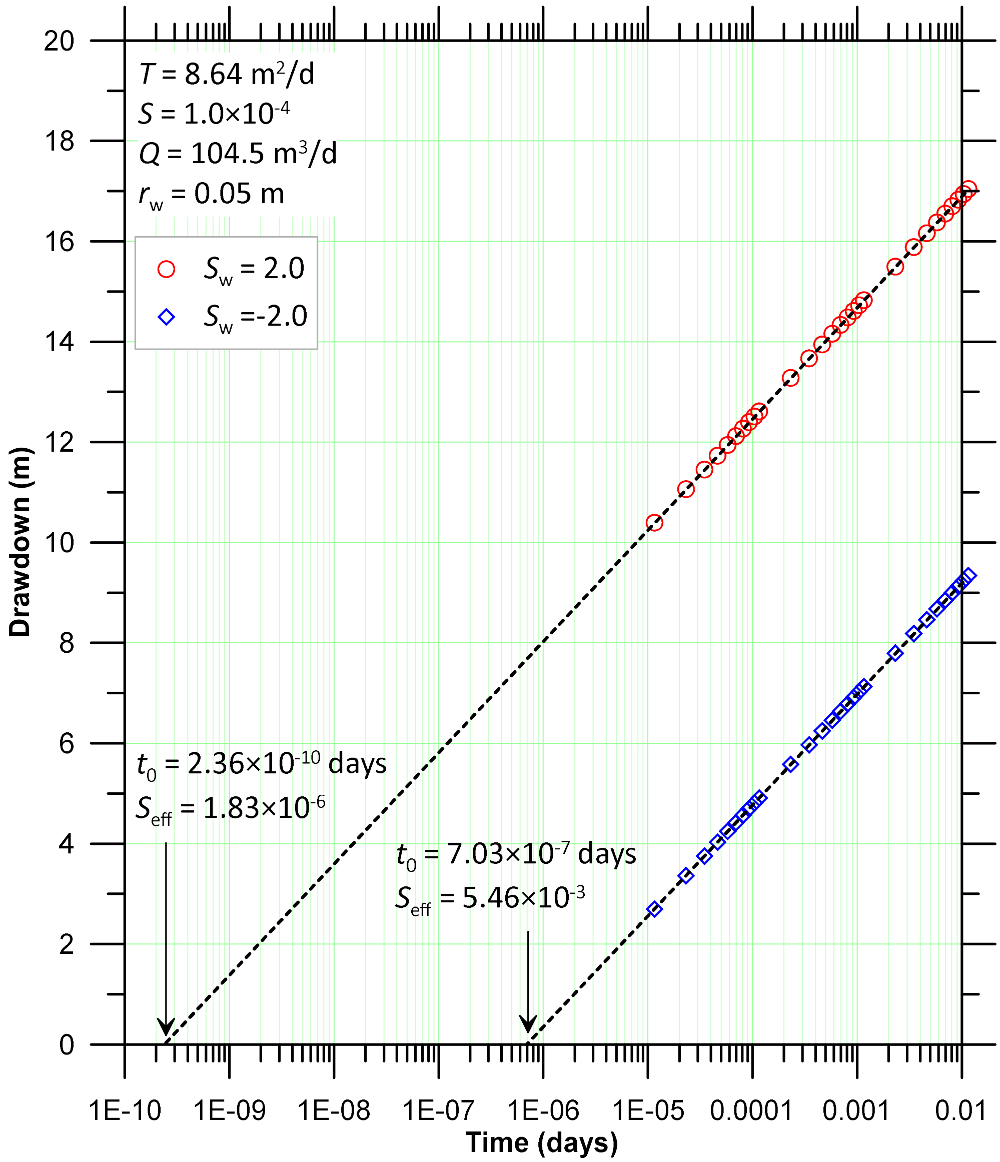


Figure 21. Example pumping well drawdowns with skin zones

11. Step tests

If the only data that are available are from the pumping well, and we want to obtain reliable estimates of the transmissivity, storativity, and well loss characteristics, we must monitor the level in the well as it is pumped at different rates. Each interval of pumping at a constant rate is referred to as a step, and this testing sequence is referred to as a *step test* (Jacob, 1946). An example dataset is shown in Figure 22.

There are two general approaches for interpreting the results of step tests:

- Steady-state analysis: the pumping well drawdowns are interpreted as if they were obtained from a sequence of steady-states; and
- Transient analysis: The entire time history of drawdowns is analyzed.

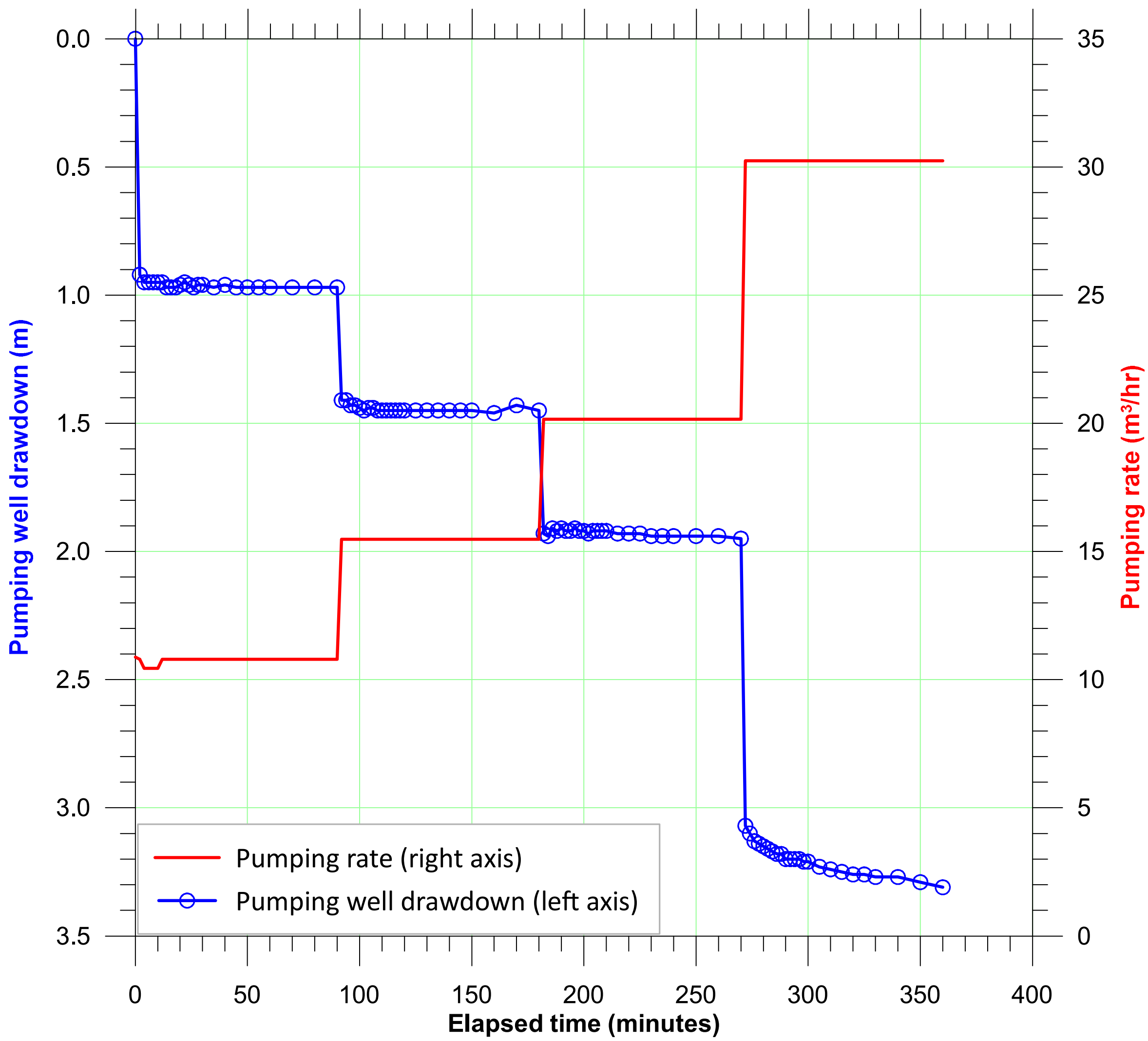


Figure 22. Example step test data

12. Interpretation of step tests: Steady-state analysis

If stabilized drawdowns in the pumping well are available for several different pumping rates, a particularly simple method is available to diagnose turbulent well losses and estimate the well loss coefficient *C*.

Since the head losses due to flow in the formation and flow across the skin zone are linear functions of the pumping rate, the total drawdown can be written as:

$$s_w = BQ + CQ^2 \qquad (29)$$

Here *B* is a lumped parameter that accounts for head losses in the formation and additional head losses across the skin zone. Following the original work of Jacob (1946), it is assumed here that the well loss exponent *P* is 2.0. Dividing both sides of Equation (29) by *Q* yields:

$$\frac{s_w}{Q} = B + CQ \qquad (30)$$

The quantity s_w/Q is referred to as the *specific drawdown*. Equation (27) predicts that if there are nonlinear well losses, the specific drawdown will increase linearly with the pumping rate.

An application of the steady-state approach is shown in Figure 23. The specific drawdown, s_w/Q is plotted against the pumping rate *Q* for each step. Figure 19 is referred to as a *Hantush-Bierschenk plot* after its initial developers Hantush (1964) and Bierschenk (1964). If the specific drawdowns approximate a straight line, we can infer:

- The slope of the line corresponds to the nonlinear well loss coefficient, *C*; and
- The intercept of the line, *B*, corresponds to the specific capacity of the well with the nonlinear well losses removed.

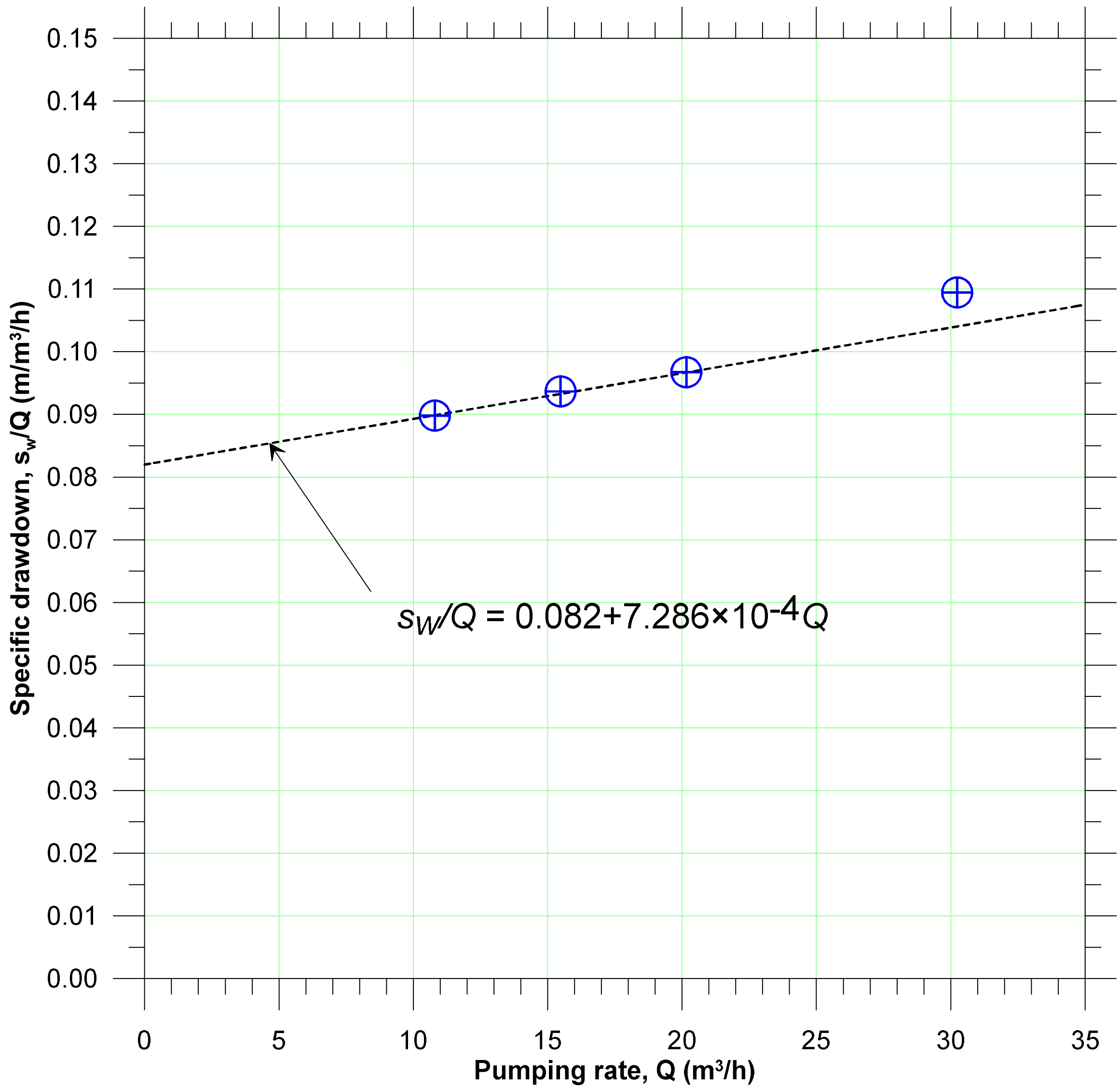


Figure 23. Hantush-Bierschenk plot, data from Figure 18

Case study: PW6/63, Guelph, Ontario – Part 1

Step tests were conducted in 1996-1998 on municipal supply wells in Guelph, Ontario, as part of a city-wide aquifer performance investigation (Jagger Hims Ltd., 1998). The discharge and drawdown data are of sufficient quality and frequency to support detailed analyses. Our analysis of the data for well PW6/63 follows a phased approach of increasing complexity:

- Diagnosis of nonlinear well losses;
- Estimation of transmissivity from the corrected specific capacity; and
- Estimation of transmissivity from a transient analysis.

The data from the step test are plotted in Figure 24.

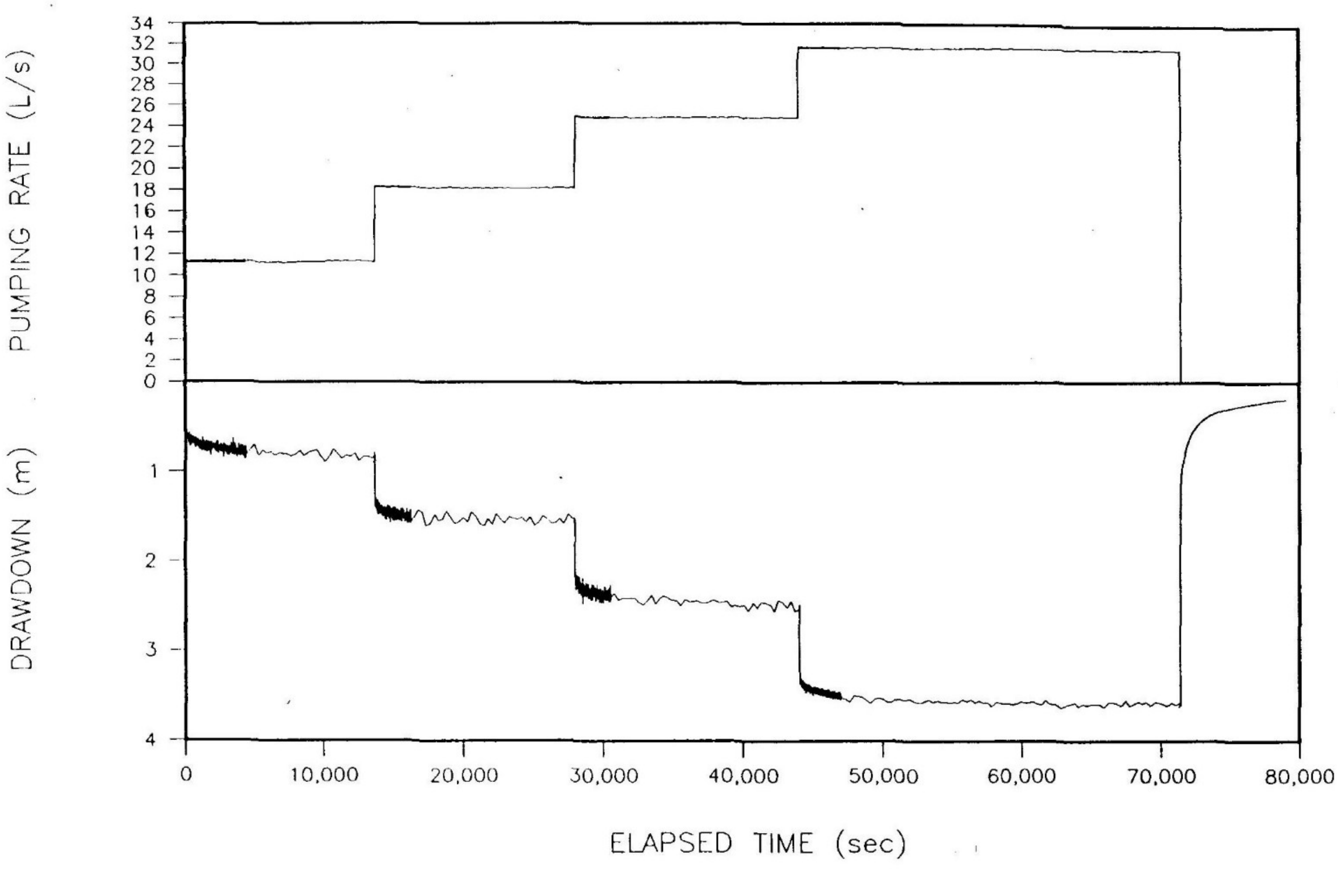


Figure 24. PW6/63 step test data

Step 1: Diagnosis of nonlinear well losses

The drawdowns appear to stabilize by the end of each pumping step. The pumping rates and drawdowns recorded at the end of each pumping step are tabulated below and plotted in Figure 25.

Step	Pumping rate, Q (L/s)	Drawdown, s_w (m)
1	11.5	0.87
2	18.6	1.53
3	25.3	2.46
4	32.0	3.60

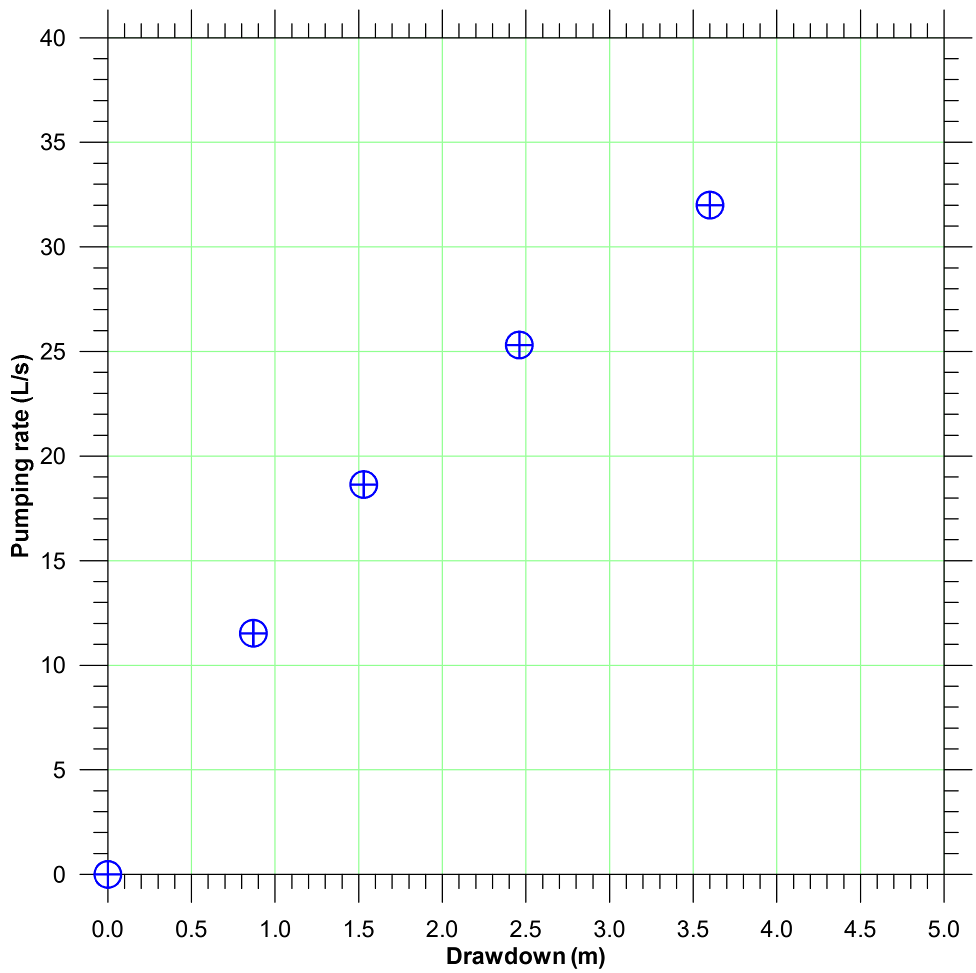


Figure 25. "Raw" specific capacity results

Under ideal conditions, that is, when all head losses are linear, the specific capacity estimated as the ratio of the pumping rate and drawdown at any particular pumping rate is identical to the slope of the relation between the pumping rate and the drawdown. However, as shown in Figure 25, during this test the specific capacity varies with the pumping rate. A reduction of the specific capacity for increased pumping rates is an initial suggestion that the pumping well drawdowns include nonlinear head losses.

To evaluate the nonlinear well losses, the results at the end of each step are assembled on a Hantush-Bierschenk plot in Figure 26.

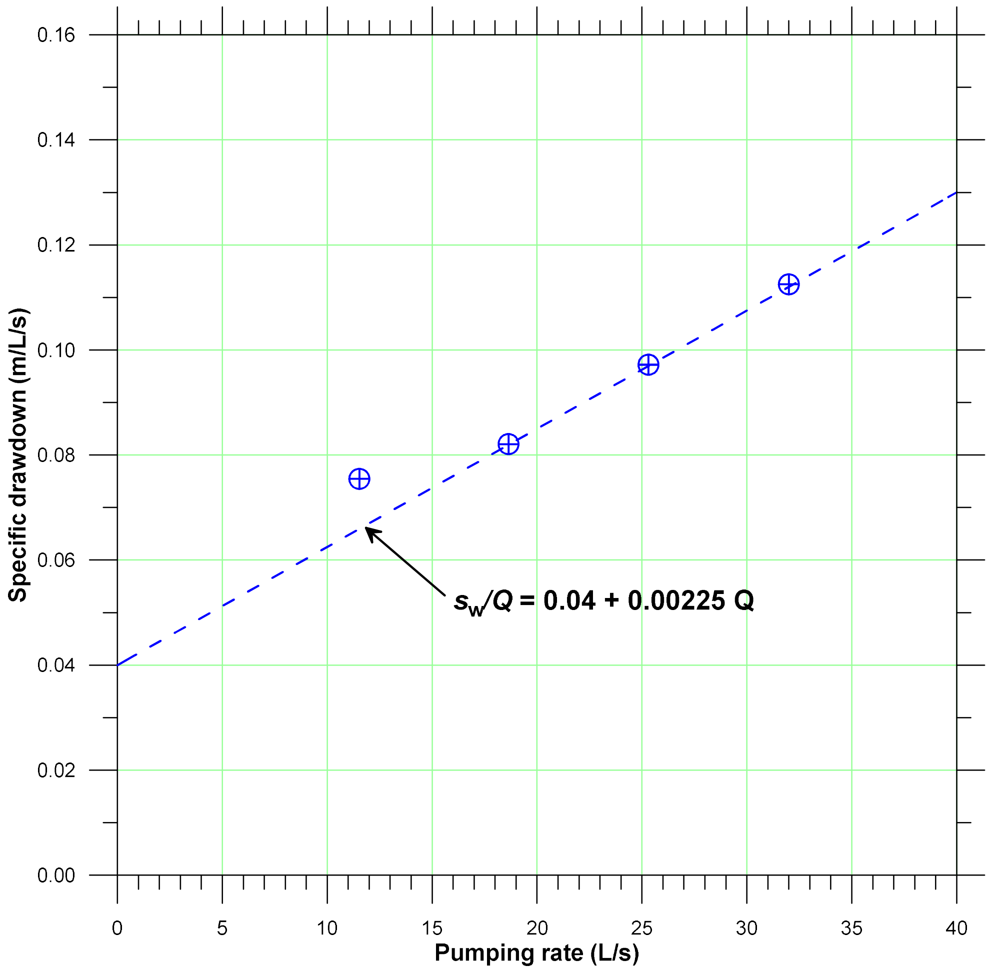


Figure 26. Specific drawdown vs. pumping rate

As shown in Figure 26, the relation between the specific drawdown and the pumping rate is nearly linear. This linearity of the plot of s_w/Q versus Q suggests that the pumping well drawdowns may be approximated using the Jacob (1946) model:

$$s_w = BQ + CQ^2$$

The parameters estimated with a linear regression analysis are:

- $\boldsymbol{B}$ **= 0.04 m/(L/s)**; and
- $\boldsymbol{C}$ **= 2.25×10^{-3} m/(L/s)^2**.

As a check, we use the fitted relation to plot the pumping rates as a function of the drawdown. The Jacob model matches the drawdowns closely (Figure 27).

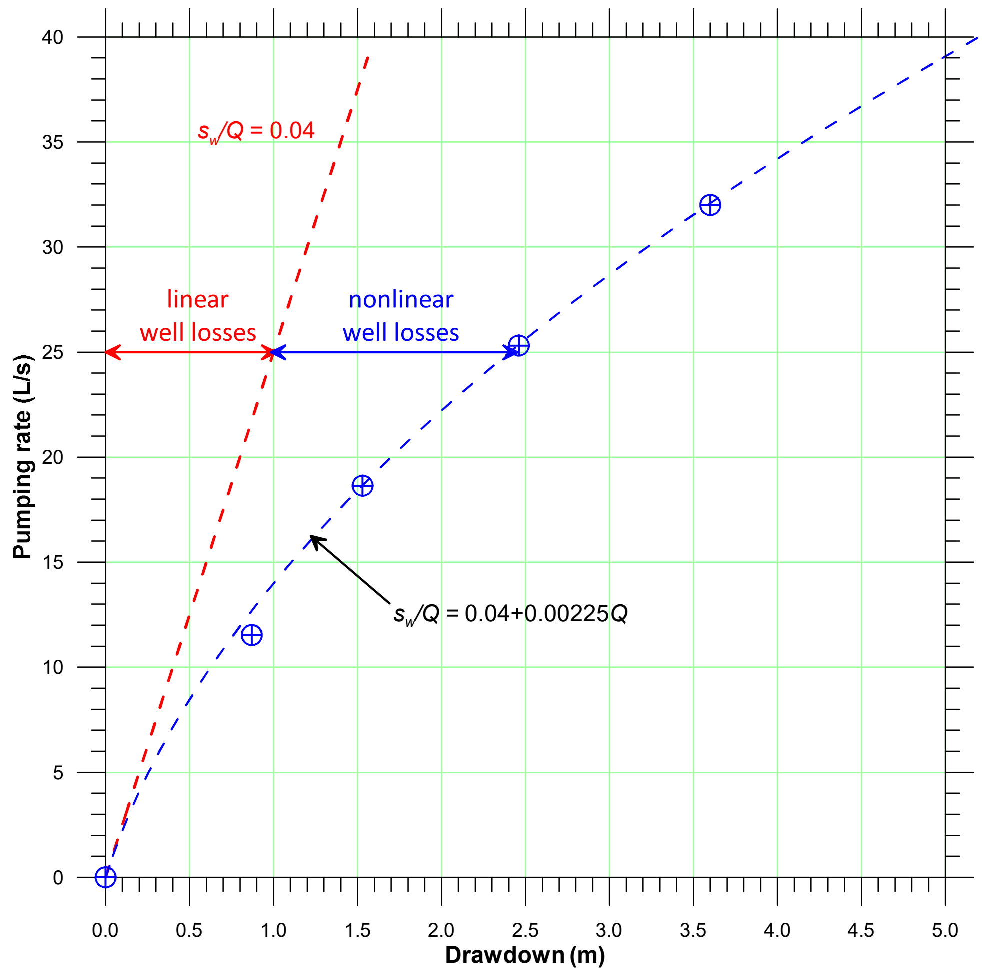


Figure 27. Specific capacity with inferred relation from the Hantush-Bierschenk plot

Step 2: Estimation of transmissivity from the "corrected" specific capacity

The coefficient *B* has an important physical interpretation. If we assume that skin losses are negligible, *B* represents the inverse of the specific capacity with the nonlinear well losses removed. A preliminary estimate of the transmissivity using the well-loss-removed specific capacity can be developed using a simple reconnaissance-level approach (Driscoll, 1986):

$$T \approx 1.4\, SC$$

The specific capacity with nonlinear well losses removed is given by:

$$SC = \frac{1}{B} = \frac{1}{0.04\ \mathrm{m/(L/s)}} = 25\ \mathrm{(L/s)/m}$$

Therefore, the transmissivity is estimated as:

$$T = 1.4\left(25\ \frac{\mathrm{L}}{\mathrm{s}}/\mathrm{m}\right) \approx 35\frac{\frac{\mathrm{L}}{\mathrm{s}}}{\mathrm{m}}\left|\frac{86{,}400\ \mathrm{s}}{\mathrm{d}}\right|\left|\frac{\mathrm{m}^3}{1000\ \mathrm{L}}\right| = \mathbf{3{,}020\ m^2/d}$$

13. Interpretation of step tests: Transient analysis

If a complete time history of drawdown is available during a step test, we can try to make use of all of the data in a transient analysis. The transient data are interpreted using the expanded form of the Theis solution. The generalization for a test in which the pumping rate varies is derived using the principle of superposition:

$$s_w(t) = \frac{2.303}{4\pi T}\sum_{i=1}^{NP(t)} \Delta Q_i \, W\left(\frac{r_w^2 S}{4T(t-ts_i)}\right) + \frac{Q_t}{4\pi T} 2S_w + CQ_t^2 \tag{31}$$

Here ts_i denotes the starting time of the i^{th} pumping step, ΔQ_i represents the change in the pumping rate at the start of this step, $NP(t)$ represents the number of steps that have occurred up to the current time t, and Q_t is the current pumping rate at time t. These terms are illustrated in Figure 28.

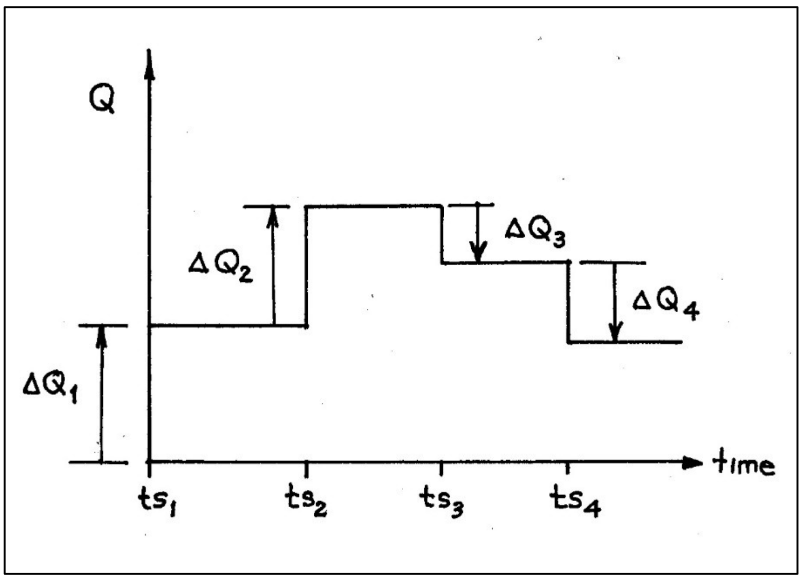


Figure 28. Schematic representation of time-varying pumping

The current pumping rate is related to the steps according to:

$$Q_t = \sum_{i=1}^{NP(t)} \Delta Q_i \tag{32}$$

In practice, the data collected during a step test are matched with Equation (28) with a fitting routine, such as the nonlinear least-squares fitting routine incorporated in packages like AQTESOLV. The application of the transient analyses will be demonstrated with a case study but first we must include a note of caution regarding the analyses.

A caution on the interpretation of step tests: Parameter correlation

In Section 3 we considered an example of a pumping well that penetrated the full thickness of an ideal aquifer. The following parameters were specified for the example:

- Transmissivity, $T = 8.64$ m^2/d;
- Storativity, $S = 10^{-4}$;
- Dimensionless skin factor, $S_w = 0.5193$; and
- Nonlinear well loss coefficient, $C = 1.340\times10^{-4}$ m^{-5}d^2.

In the previous calculation, we assumed that the well was pumped at a constant rate. This time, let us assume that the well is pumped for three even steps according to the following schedule:

Elapsed time	Pumping rate, m^3/day
0 to 60 minutes	34.848
60-120 minutes	69.696
120-180 minutes	104.544

The drawdowns at the pumping well calculated with Equation (28) are plotted in Figure 29.

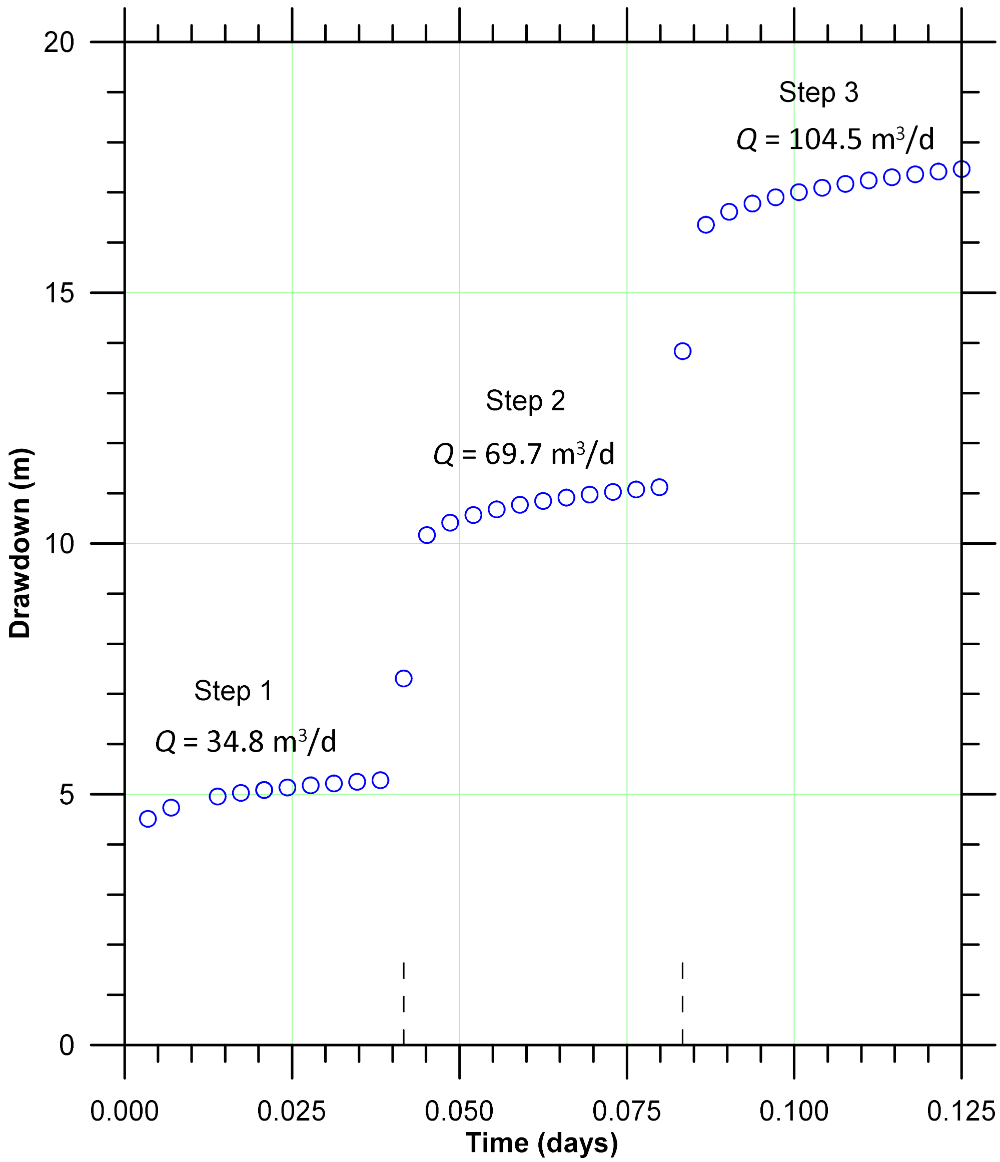


Figure 29. Pumping well drawdowns for a hypothetical step test

The analysis package AQTESOLV is used to fit the full transient record with Equation (25). To use a model that incorporates well losses, we must do two things with AQTESOLV:

- Tell AQTESOLV that we are interested in interpreting the drawdown as if it came from a pumping well and not an observation well. We do this by specifying the pumping well as an observation well at zero radial distance from the pumping well; and
- Choose the "Confined – Theis (1935) step drawdown test" solution.

How well does AQTESOLV do when we ask it to estimate simultaneously the transmissivity and storativity, T and S, *and* the well loss parameters C and S_w? The results of the automatic fit are shown in Figure 30.

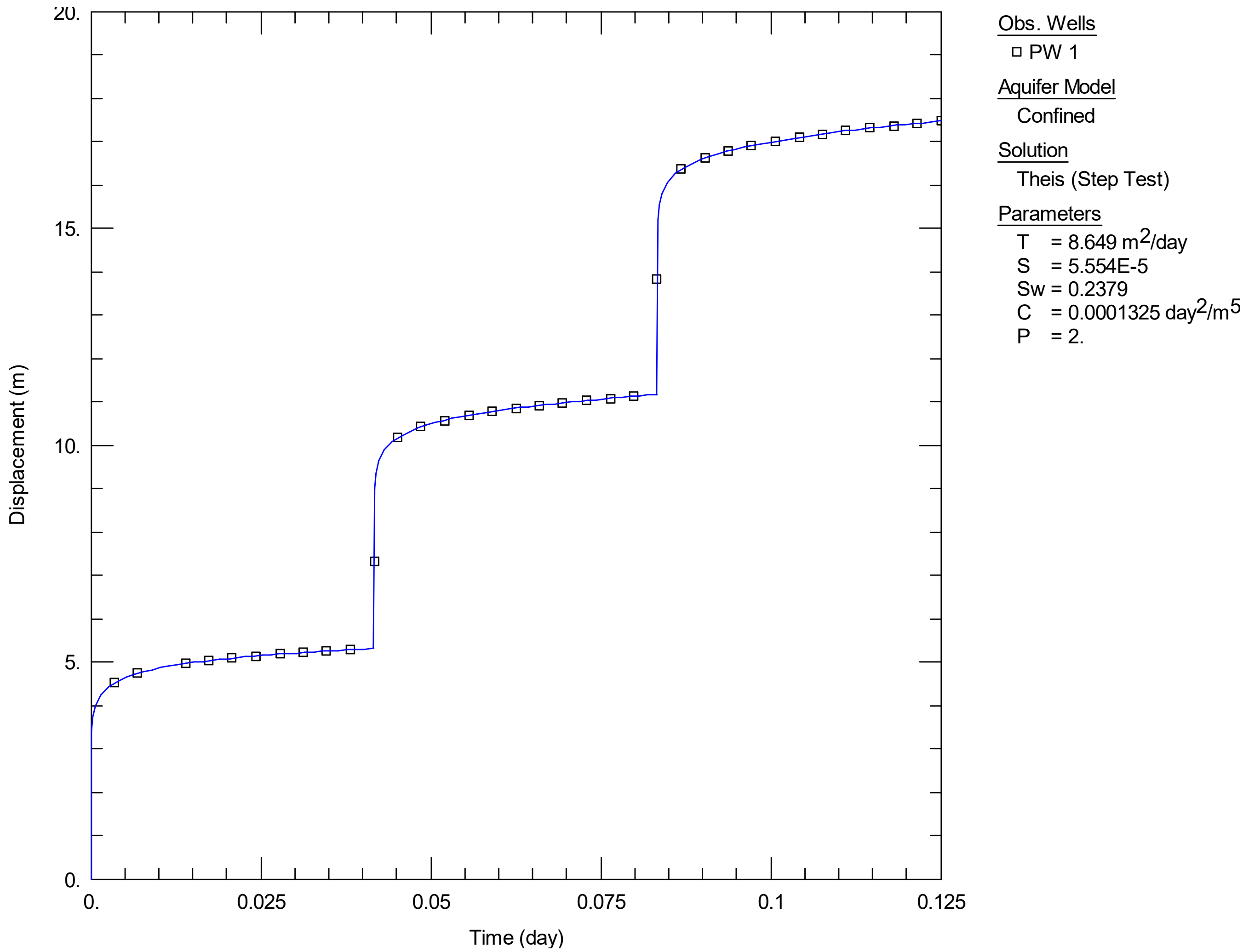


Figure 30. AQTESOLV match to hypothetical step test results

As shown in Figure 15, the drawdown data are matched very closely. The specified and fitted parameter values are listed below.

Parameter	Specified value	Fitted value
T	8.64 m^2/d	8.65 m^2/d
S	1.0×10^{-4}	5.55×10^{-5}
S_w	0.5193	0.2379
C	1.34×10^{-4} d^2/m^5	1.32×10^{-4} d^2/m^5
P	2.0 (fixed)	2.0 (fixed)

The fitted values of the transmissivity, *T*, and the nonlinear well loss coefficient, *C*, are very close to the values that were specified in the calculations. By pumping the well at more than one pumping rate, we increase our chances of obtaining unique estimates for these parameters. However, the storativity, *S*, and the skin loss coefficient, S_w, are significantly different from the specified values.

Why do we obtain an essentially perfect match to the drawdowns but with very different parameter values? Is it possible that the parameters estimated through the "objective" nonlinear least-squares fitting are not unique?

To assess whether the parameter values estimated for a particular analysis are unique, it is necessary to examine whether any of the fitted parameters are correlated. With the AQTESOLV software it is possible to examine parameter correlation, under the window labeled **diagnostics**. The report of the fitting for this example is reproduced in Figure 31. Reviewing the reported "Parameter Correlations", we see that the storage coefficient and the skin loss coefficient have Parameter Correlation values of 1.00. This means that the parameters are perfectly correlated.

DIAGNOSTICS REPORT

Diagnostic Statistics

Estimation complete! Parameter change criterion (ETOL) reached.

Aquifer Model: Confined
Solution Method: Theis (Step Test)

Estimated Parameters

Parameter	Estimate	Std. Error	Approx. C.I.	t-Ratio	
T	8.649	0.004118	+/- 0.008397	2100.2	m^2/day
S	5.554E-5	4.83E-5	+/- 9.848E-5	1.15	
Sw	0.2379	0.4298	+/- 0.8763	0.5536	
C	0.0001325	1.319E-7	+/- 2.689E-7	1004.7	day^2/m^5
P	2.	not estimated			

C.I. is approximate 95% confidence interval for parameter
t-ratio = estimate/std. error
No estimation window

K = T/b = 0.8649 m/day (0.001001 cm/sec)
Ss = S/b = 5.554E-6 1/m

Parameter Correlations

	T	S	Sw	C
T	1.00	-0.85	-0.85	0.31
S	-0.85	1.00	1.00	-0.21
Sw	-0.85	1.00	1.00	-0.21
C	0.31	-0.21	-0.21	1.00

Residual Statistics

for weighted residuals

Sum of Squares . . . . 4.392E-5 m^2
Variance . . . . 1.417E-6 m^2
Std. Deviation . . . . 0.00119 m
Mean . . . . -0.0001331 m
No. of Residuals 35
No. of Estimates 4

Figure 31. Diagnostic reports for the step test example

What does a Parameter Correlation value of 1.00 mean? In this case, it means that the storativity *S* and the dimensionless skin factor S_w are perfectly correlated. From the perspective of curve fitting, this means that the values of *S* and S_w *cannot* be estimated independently; that is, it is impossible to obtain unique estimates of *S* and S_w. As shown in Figure 32, with *S* fixed at two different values equally good matches to the observations are obtained with two different values of S_w.

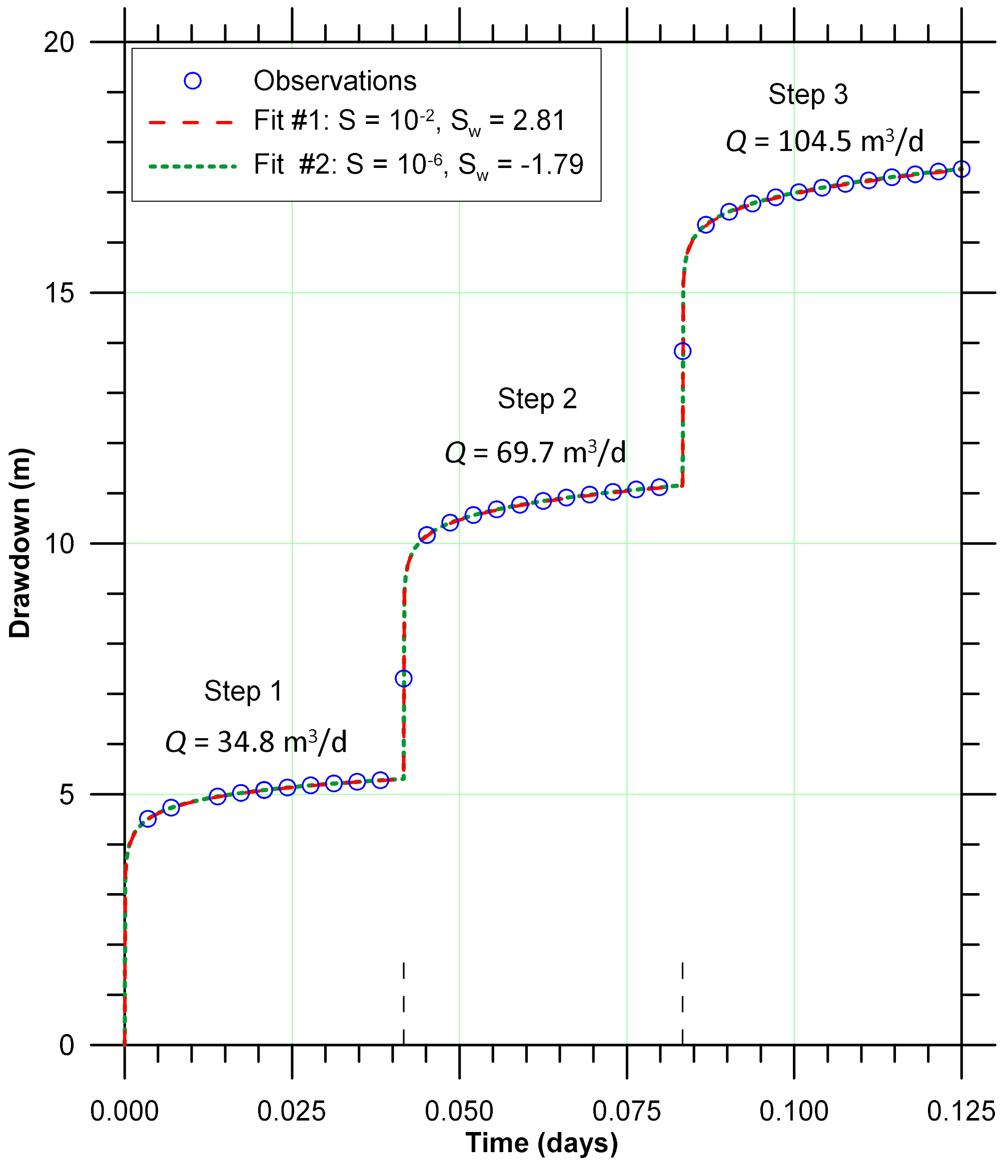


Figure 32. Alternative AQTESOLV matches to hypothetical step test results

To understand why the storativity and skin factor are perfectly correlated we return to the Cooper-Jacob solution with the addition of skin losses:

$$s(r_w, t) = s_{formation}(r_w, t) + \Delta s_{skin}$$

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2 S}{4Tt}\right\}\right] + \frac{Q}{4\pi T}\, 2S_w$$

Making use of the properties of the log function, the solution can be re-arranged as:

$$s(r_w, t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S}{4Tt}\right\} - \ln\{EXP\{2S_w\}\}\right]$$

Collecting terms:

$$s(r_w, t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S\; EXP\,\{-2S_w\}}{4Tt}\right\}\right] \qquad (33)$$

Defining an "effective" storage coefficient, S_E, as:

$$S_E = SEXP\{-2S_w\} \qquad (34)$$

Equation (33) reduces to the Cooper-Jacob approximation with the actual storage coefficient S replaced by S_E:

$$s(r_w, t) = \frac{Q}{4\pi}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S_E}{4Tt}\right\}\right] \qquad (35)$$

What do the paired values of S and S_w in Figure 28 have in common? For the original parameters and the parameter values shown in Figure 28, we calculate:

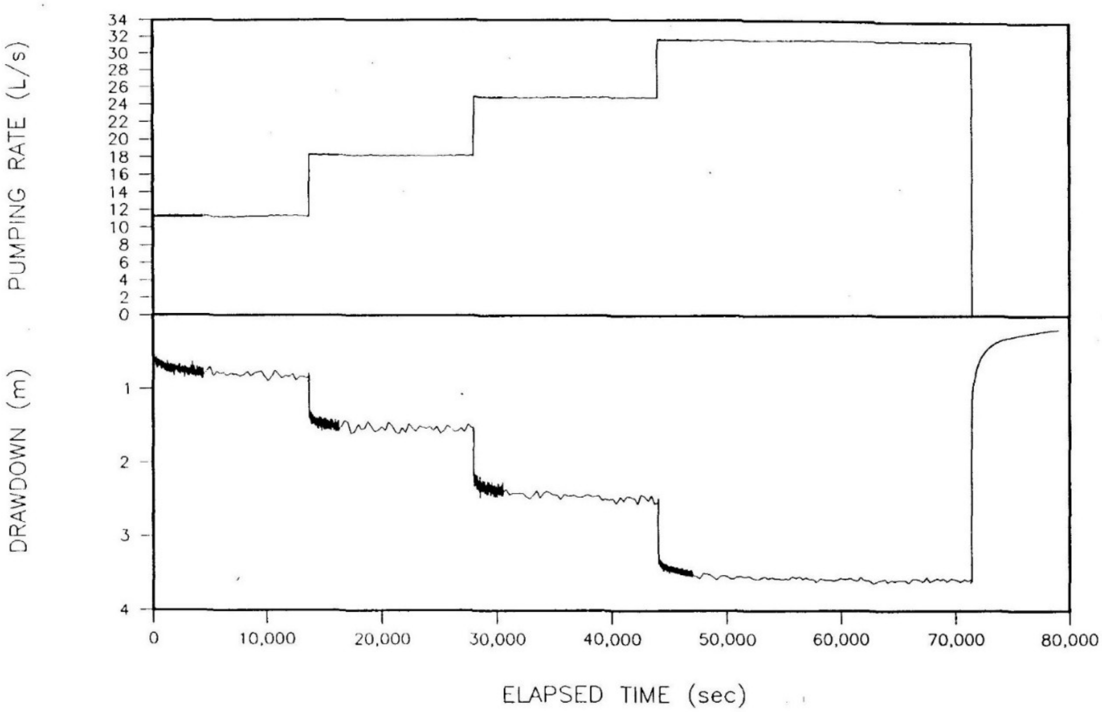
$$\begin{aligned} S \times EXP\{-2S_w\} &= (10^{-2}) \times EXP\{-2(2.81)\} = 3.6 \times 10^{-5} \\ &= (10^{-6}) \times EXP\{-2(-1.79)\} = 3.6 \times 10^{-5} \end{aligned}$$

The paired values of S and S_w yield identical values of $S \times EXP\{-2S_w\}$. These results demonstrate that it is possible to only know the product $S \times EXP\{-2S_w\}$.

Confined storage coefficients vary over a relatively narrow range, from about 1.0×10^{-6} to 1.0×10^{-4}. Fitted values that are well outside of this range likely suggest the presence of a skin zone around the pumping well.

Case study: PW6/63, Guelph, Ontario – Part 2

The plot of the pumping history and drawdowns for the PW6/63 step test is reproduced below. The data are of sufficient quality to support a more rigorous transient analysis. With the results of the rigorous analyses in hand we can evaluate the reliability of the transmissivity estimates developed with the simpler approaches.



A computer-assisted analysis package is used to estimate the test data, matching the Theis solution to the drawdowns. The following decisions are made to constrain the analysis and avoid the non-uniqueness arising from parameter correlation:

- We assume a "physically realistic" value for the storage coefficient, *S*; of 1.0×10^{-5}; and
- We fix the value of the nonlinear well loss coefficient, *C*, based on the results from the Hantush-Bierschenk plot.

$$C = 0.00225 \ \frac{m}{(\mathrm{L/s})^2} \left| \frac{1000 \ L}{m^3} \right|^2 = 2{,}250 \ \frac{m}{(m^2/\mathrm{s})}$$

The results of the computer-assisted analysis are shown in Figure 33. As shown in the figure, it is possible to match closely the entire time-drawdown record with a transmissivity of **3,760 m²/d**.

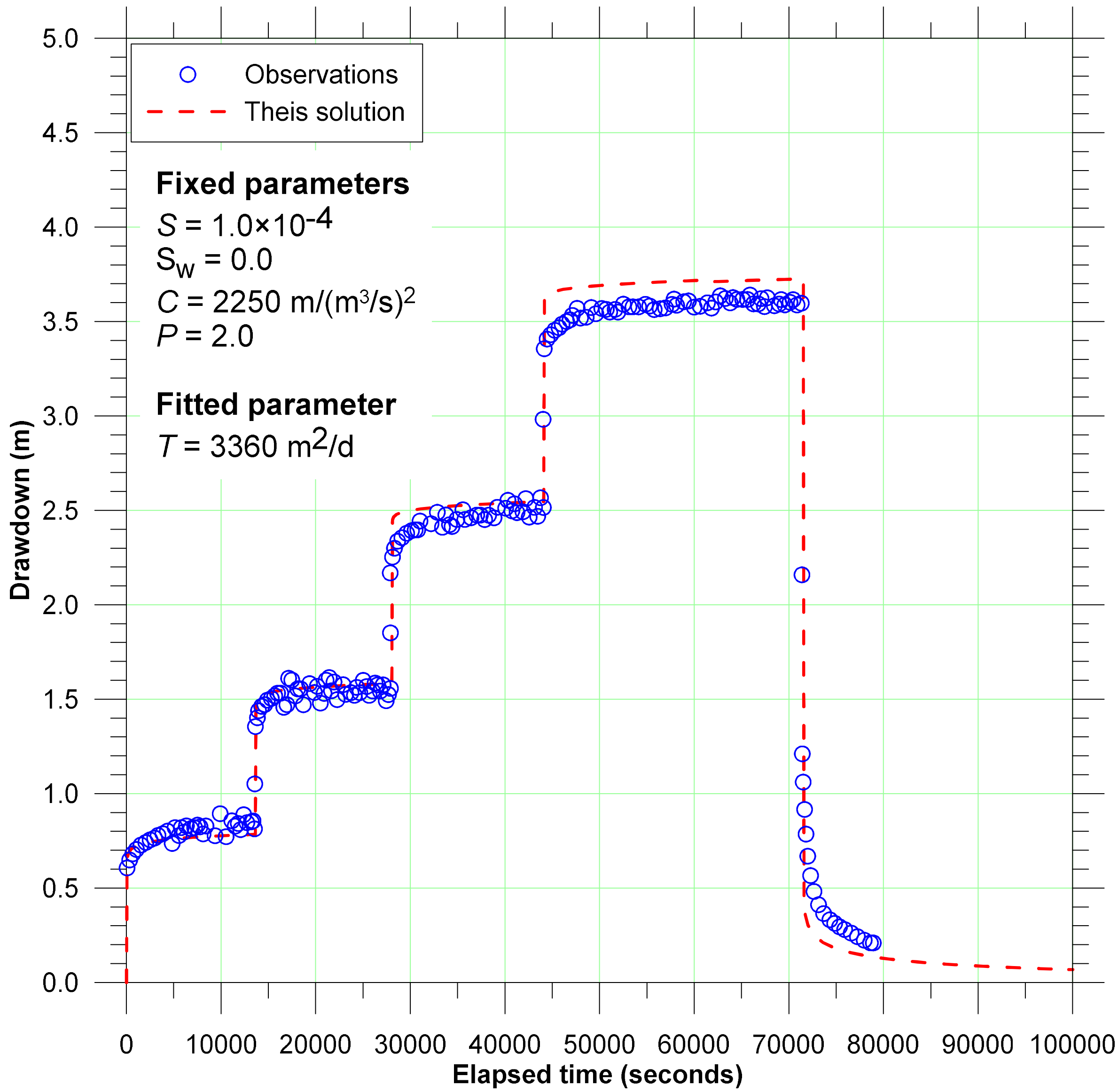


Figure 33. Match to the full transient drawdown record

Assessment of the analysis of the PW6/63 step test

The advantage of estimating the transmissivity with the corrected specific capacity is that it is simple to do. Is the resulting estimate consistent with the estimate developed from a more rigorous analysis that considers all of the available data?

- Correlation with specific capacity: $\boldsymbol{T} \cong$ **3,020 m²/d**; and
- Rigorous transient analysis: $\boldsymbol{T} =$ **3,760 m²/d**.

The transmissivity estimate obtained with the corrected specific capacity is relatively close to the estimate developed from the rigorous analysis. It is certainly consistent with our expectations regarding a "reconnaissance-level" estimate. The results of the simplified and rigorous analyses are in fact complementary. The preliminary estimate of the transmissivity derived with the estimate of the specific capacity with the nonlinear losses removed provides a useful check on our more rigorous analysis.

The results of the analyses highlight the importance of accounting for well losses in this case. The "raw" specific capacities range from 13.2 to 8.9 L/s/m. These are significantly smaller than the value estimated after the nonlinear well losses are removed, 25 L/s/m.

14. Synthesis of pumping well and observation well drawdowns

The consistency of the drawdowns for a pumping well and observation wells can best be assessed by plotting all of the drawdown data on a semilog *composite plot* (Cooper and Jacob, 1946). The composite plot has an axis of time/radius^2. According to the Theis conceptual model, the drawdowns at any point in the pumped aquifer will fall on a single line on a composite plot. However, we know that for most pumping wells there are head losses in addition to those that occur in the formation. If we assemble the data on a composite semilog plot the additional head losses in the pumping well will plot with a constant offset with respect to the observation wells. This concept is illustrated with some example calculations.

Let us consider a simple example involving a single pumping well that penetrates the full thickness of a homogeneous, horizontal, and perfectly confined aquifer of infinite extent. The aquifer is assumed to have a transmissivity and storativity of 8.64 m^2/day and 1.0×10^{-4}, respectively. The aquifer is pumped at a constant rate of 104.54 m^3/day. The observation well is located 10 m from the pumping well. The well is surrounded by a skin and there are nonlinear well losses. The additional well losses are characterized by the following parameters:

- S_w = 0.5193; and
- P = 2; and
- $C = 1.340\times10^{-4}$ m^{-5}d^2.

The results of the example calculations are assembled on a composite plot in Figure 34. The results do not approximate a single straight line. The drawdowns for the pumping well and observation appear to approximate parallel lines. The same slopes and offset of the pumping well and observation well drawdowns in Figure 34 are key diagnostic results. The parallel lines confirm that it is possible to estimate a representative bulk average transmissivity. The offset points to additional well losses in the pumping well drawdowns.

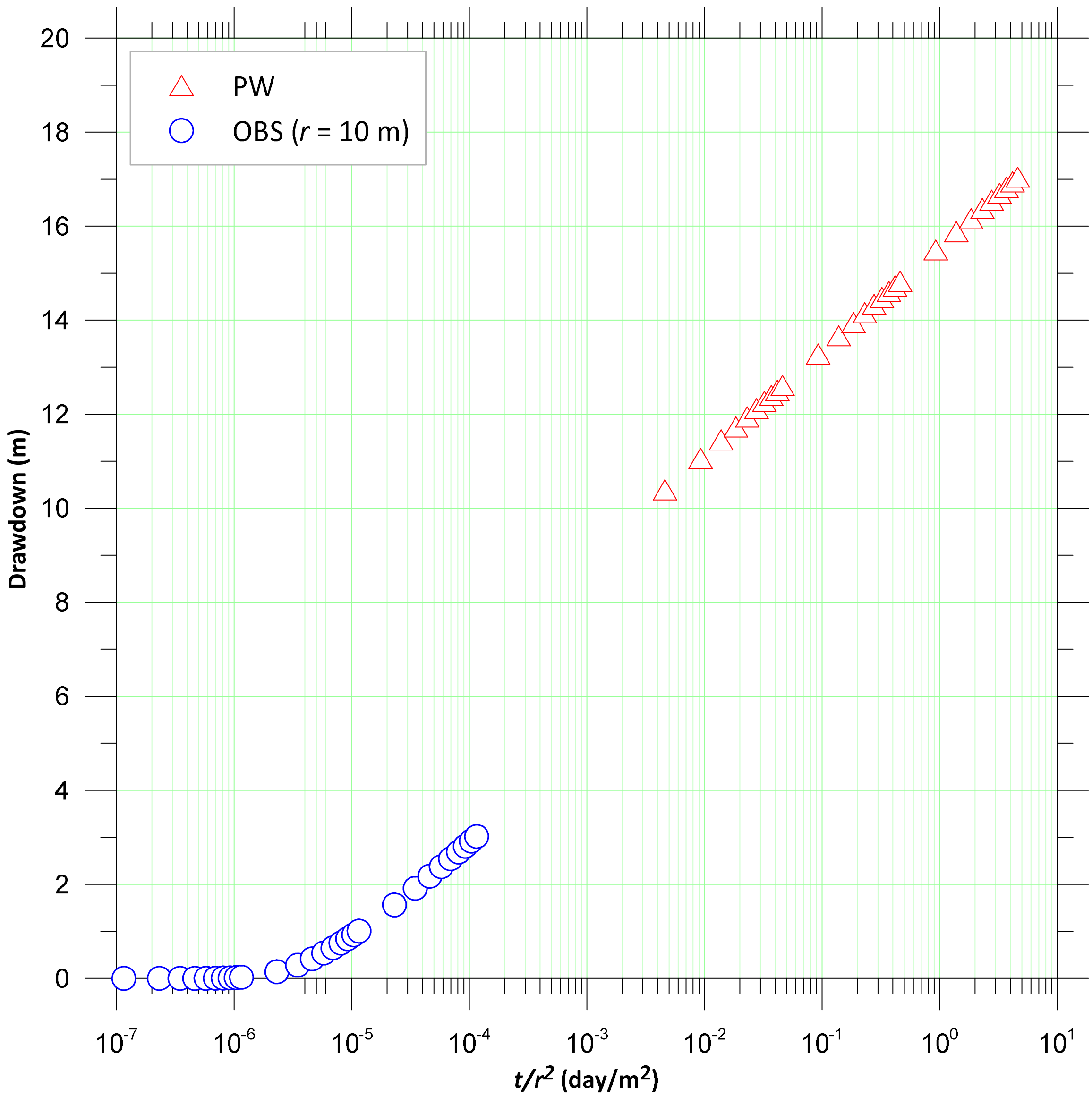


Figure 34. Composite plot for the pumping well and observation well

The Cooper-Jacob straight-line analyses are shown in Figure 35. The parallel slopes yield a consistent estimate of the transmissivity. Since the storage coefficient is estimated from the intercept of the straight lines fitted through the data, another important outcome of this plotting approach is that the Cooper-Jacob straight-line analyses of the individual records will yield different storage coefficients.

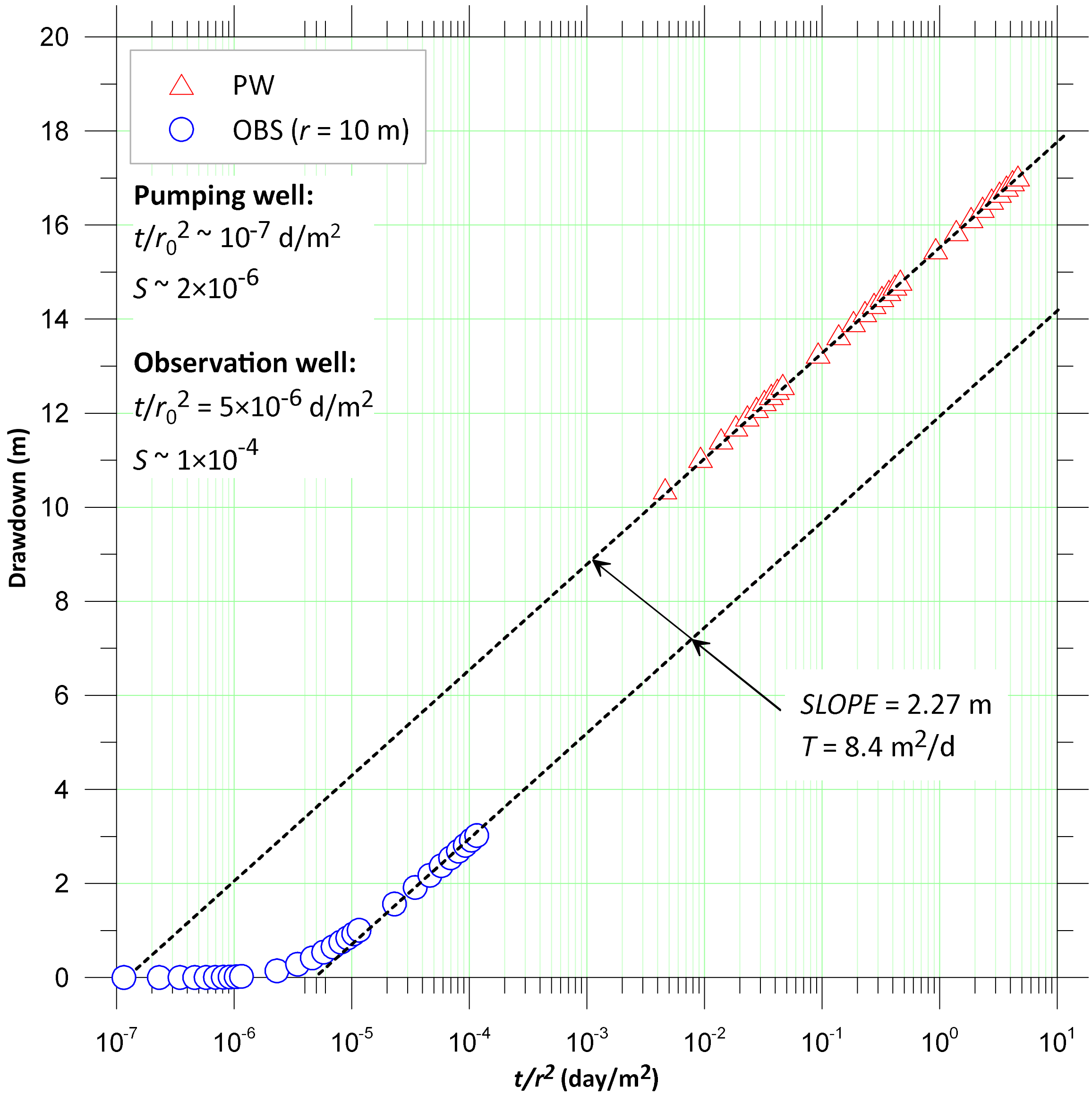


Figure 35. Cooper-Jacob analyses on the composite plot

As shown in Figure 35, the storage coefficient estimated for the observation well is 1.0×10^{-4} (as specified) and the storage coefficient for the pumping well is estimated to be 2.0×10^{-6}. The Cooper-Jacob solution can be used to explain why the data from the pumping well yields an inconsistent storage coefficient. If we replace the Theis well function with the Cooper-Jacob approximation, we obtain:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right] + CQ^P + \frac{Q}{4\pi T}2S_w$$

Expanding the *ln* term and rearranging yields:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2}{4Tt}\right\} - ln\{S\} + 2S_w + \frac{4\pi T}{Q}CQ^P\right]$$

Expanding using the properties of the log function yields:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2}{4Tt}\right\} - ln\left\{S \times EXP\{-2S_w\} \times EXP\left\{-\frac{4\pi T}{Q}CQ^P\right\}\right\}\right]$$

We can write this in an equivalent form in terms of an "effective" storage coefficient as:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2 S_{eff}}{4T}\right\}\right] \quad (35)$$

with:

$$S_{\text{eff}} = S \times EXP\{-2S_w\} \times EXP\left\{-\frac{4\pi T}{Q}CQ^P\right\} \quad (36)$$

Turbulence and skin effects confound estimation of reliable storativities from pumping well data. However, the inference of different storage coefficients for observation and pumping wells is actually useful. An inconsistent, and in some cases non-physical, estimate of the storage coefficient from the pumping well drawdowns is an important indicator that there are additional head losses in the pumping well.

15. Key points

1. Head loss in the formation is only one of the processes that cause drawdowns in a pumping well.

2. If all we have is one point (Q,s_W) and we don't know much about the well, then estimating the transmissivity from the "raw" specific capacity may be the best we can do. This approach does not allow us to consider specific information we may have, such as the duration of pumping and well construction details, and the significance of well losses.
3. If a time history of pumping well drawdowns is available, and our only objective is to estimate transmissivity, we can begin and end with the Cooper-Jacob straight-line analysis.
4. If we want to know more about a pumping well than just the transmissivity in its vicinity, we require both a constant-rate pumping test and a step test. Step testing is the only definitive method of evaluating nonlinear well losses.
5. When interpreting pumping well drawdowns, we usually cannot estimate all parameter values. For example, we cannot estimate the storage coefficient and skin factor separately. Watch for non-physical parameter estimates and parameter correlation.

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